

Approved by Omega Math July 28, 2014

High School (9-12)

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in fourth credit courses or advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories including Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability, and Contemporary Mathematics.

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (\star).

Number and Quantity	Geometry
The Real Number System (N-RN)	Congruence (G-CO)
Quantities (N-Q)	 Similarity, Right Triangles, and Trigonometry (G-SRT)
The Complex Number System (N-CN)	Circles (G-C)
 Vector and Matrix Quantities (N-VM) 	 Expressing Geometric Properties with Equations (G-GPE)
	 Geometric Measurement and Dimension (G-GMD)
Algebra	 Modeling with Geometry (G-MG)
 Seeing Structure in Expressions (A-SSE) 	
Arithmetic with Polynomials and Rational Expressions (A-APR)	Modeling
Creating Equations (A-CED)	
 Reasoning with Equations and Inequalities (A-REI) 	Statistics and Probability
	 Interpreting Categorical and Quantitative Data (S-ID)
Functions	 Making Inferences and Justifying Conclusions (S-IC)
 Interpreting Functions (F-IF) 	 Conditional Probability and the Rules of Probability (S-CP)
Building Functions (F-BF)	Using Probability to Make Decisions (S-MD)
 Linear, Quadratic, and Exponential Models (F-LE) 	
Trigonometric Functions (F-TF)	Contemporary Mathematics
	Discrete Mathematics (CM-DM)
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#### High School - Number and Quantity Overview

#### The Real Number System (N-RN)

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

#### Quantities (N-Q)

• Reason quantitatively and use units to solve problems

#### The Complex Number System (N-CN)

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

#### Vector and Matrix Quantities (N-VM)

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

#### **Mathematical Practices (MP)**

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

#### **High School - Number and Quantity**

**Numbers and Number Systems.** During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3.... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that  $(5^{1/3})^3$  should be  $5^{(1/3)3} = 5^1 = 5$  and that  $5^{1/3}$  should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

**Quantities.** In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity: The Real Number System (N-RN) Extend the properties of exponents to rational exponents.			
<u>Standards</u>	Labe	Mathematical Practices	Explanations and Examples
Students are expected to:	<u>-</u>		
HS.N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)^3}$ to hold, so $(5^{1/3})^3$ must equal 5.	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others.	Students may explain orally or in written format.
Connections: 11-12.RST.4; 11-12.RST.9; 11-12.WHST.2d Intermediate Algebra 5.4			
HS.N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. Intermediate Algebra 5.1	*	HS.MP.7. Look for and make use of structure.	Examples: • $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$ ; $5^{\frac{2}{3}} = \sqrt[3]{5^2}$ • Rewrite using fractional exponents: $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$ • Rewrite $\frac{\sqrt{x}}{x^2}$ in at least three alternate forms. Solution: $x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{\sqrt{x^3}} = \frac{1}{x\sqrt{x}}$ • Rewrite $\sqrt[4]{2^{-4}}$ .using only rational exponents. • Rewrite $\sqrt[3]{x^3 + 3x^2 + 3x + 1}$ in simplest form.

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Number and Quantity: The Real Number System (N-RN)			
Use properties of rational an	d irration	al numbers.	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.N-RN.3. Explain why the sum or product of two rational numbers are rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Connection: 9-10.WHST.1e Intermediate Algebra 1.1	9-10	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others.	<ul> <li>Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between multiplication and addition).</li> <li>Example: <ul> <li>Explain why the number 2π must be irrational, given that π is irrational. Answer: if 2π were rational, then half of 2π would also be rational, so π would have to be rational as well.</li> </ul> </li> </ul>

Number and Quantity: Quantities * (N-Q)			
Reason quantitatively and us	se units t	o solve problems.	Explanations and Examples
Students are expected to:		Mathematical Fractices	
HS.N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Connections: SCHS-S1C4-02; SSHS-S5C5-01 Basic Math/Pre Algebra 6.4 Elementary Algebra 10.3 Intermediate Algebra 7.4-7.6	9-10 + ★	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.	Include word problems where quantities are given in different units, which must be converted to make sense of the problem. For example, a problem might have an object moving12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour: 24000 sec • $\frac{1\min}{60 \sec}$ • $\frac{1hr}{60\min}$ • $\frac{1day}{24hr}$ which is more than 8 miles per hour. Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs.
HS.N-Q.2. Define appropriate quantities for the purpose of descriptive modeling. Connection: SSHS-S5C5-01 Basic Math/Pre Algebra 8.1 Elementary Algebra 10.3 Intermediate Algebra 6 1	9-10 + ★	HS.MP.4. Model with mathematics. HS.MP.6. Attend to precision.	<ul> <li>Examples:</li> <li>What type of measurements would one use to determine their income and expenses for one month?</li> <li>How could one express the number of accidents in Arizona?</li> </ul>

Number and Quantity: Quantities * (N-Q) Reason guantitatively and use units to solve problems.			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. Basic Math/Pre Algebra 3.4	9-10 ★	HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.	<ul> <li>The margin of error and tolerance limit varies according to the measure, tool used, and context.</li> <li>Example: <ul> <li>Determining price of gas by estimating to the nearest cent is appropriate because you will not pay in fractions of a cent but the cost of gas is \$3.479/gallon.</li> </ul> </li> </ul>

Number and Quantity: The Complex Number System (N-CN)			
Perform arithmetic operations with complex numbers.			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.N-CN.1. Know there is a complex number <i>i</i> such that $\hat{f} =$ -1, and every complex number has the form <i>a</i> + <i>bi</i> with <i>a</i> and <i>b</i> real. Intermediate Algebra 6.2 College Algebra 1.3 Trigonometry 8.1	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.6. Attend to precision.	
HS.N-CN.2. Use the relation $\vec{r} = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.	<ul> <li>Example:</li> <li>Simplify the following expression. Justify each step using the commutative, associative and distributive properties. (3-2i)(-7+4i)</li> </ul>
Connection: 11-12.RST.4			Solutions may vary; one solution follows:
Intermediate Algebra 6.2 College Algebra 1.3 Trigonometry 8.1			$(3-2i)(-7+4i)$ $3(-7+4i)-2i(-7+4i)$ Distributive Property $-21+12i+14i-8i^{2}$ Distributive Property $-21+(12i+14i)-8i^{2}$ Associative Property $-21+i(12+14)-8i^{2}$ Distributive Property $-21+26i-8i^{2}$ Computation $-21+26i-8(-1)$ $i^{2}=-1$ $-21+26i+8$ Computation $-21+8+26i$ Computative Property $-13+26i$ Computation

Number and Quantity: The C	omplex Nu	Imber System (N-CN)	
Perform arithmetic operations		ex numbers.	Furtherships and Evenues
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to: HS.N-CN.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. Connection: 11-12.RST.3 Intermediate Algebra 6.2 College Algebra 1.3 Trigonometry 8.1		HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.	Example: • Given $w = 2 - 5i$ and $z = 3 + 4i$ a. Use the conjugate to find the modulus of w. b. Find the quotient of z and w. Solution: a. $ w ^2 = w\overline{w}$ $ w ^2 = 4 + 0i - 10i - 25i^2$ $ w ^2 = 4 - 25i^2$ $ w ^2 = 4 - 25(-1)$ $ w ^2 = 4 - 25(-1)$ $ w ^2 = 4 + 25$ $ w ^2 = 29$ $ w  = \sqrt{29}$ b. $\frac{z}{w} = \frac{3 + 4i}{2 - 5i}$ $\frac{z}{w} = \frac{3 + 4i}{2 - 5i} (\frac{2 + 5i}{2 + 5i})$ $\frac{z}{w} = \frac{6 + 15i + 8i - 20}{4 + 25}$ $\frac{z}{w} = \frac{-14 + 23i}{29}$

Number and Quantity: The Complex Number System (N-CN)			
Represent complex numbers	and the	ir operations on the complex p	plane.
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.N-CN.4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. Connection: 11-12.RST.3 <b>Trigonometry 8.2</b>	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.	<ul> <li>Students will represent complex numbers using rectangular and polar coordinates.</li> <li>imaginary <ul> <li>a + bi = r(cos θ + sin θ)</li> <li>bi a + bi</li> <li>bi</li></ul></li></ul>
HS.N-CN.5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3} i)^3 = 8$ because $(-1 + \sqrt{3} i)$ has modulus 2 and argument 120°. <b>Trigonometry 8.2</b>	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.	

Number and Quantity: The	Number and Quantity: The Complex Number System (N-CN)			
Represent complex numbers	s and the	ir operations on the complex	plane.	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.N-CN.6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	*	HS.MP.2. Reason abstractly and quantitatively.		
Connection: 11-12.RST.3				
Trigonometry 8.2				

Number and Quantity: The Complex Number System (N-CN)			
Use complex numbers in pol	Use complex numbers in polynomial identities and equations.		
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.	*		<ul> <li>Examples:</li> <li>Within which number system can x² = - 2 be solved? Explain how you know.</li> <li>Solve x² + 2x + 2 = 0 over the complex numbers.</li> <li>Find all solutions of 2x² + 5 = 2x and express them in the form a + bi.</li> </ul>
Elementary Algebra 9.3 Intermediate Algebra 6.7 College Algebra 1.4			
US N CN 8 Extend	.*.	HS MP 7 Look for and make	
polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as (x + 2i)(x - 2i).	**	use of structure.	
US N CN 9, Know the	.*.	HS MP 2 Construct viable	Examples:
Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Connection: 11-12.WHST.1c College Algebra 3.5	**	arguments and critique the reasoning of others. HS.MP.7. Look for and make use of structure.	<ul> <li>How many zeros does -2x²+3x-8 have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra.</li> <li>How many complex zeros does the following polynomial have? How do you know?</li> <li>p(x)=(x²-3)(x²+2)(x-3)(2x-1)</li> </ul>

Number and Quantity: Vector and Matrix Quantities (N-VM)			
Represent and model with ve	ector qua	intities.	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.N-VM.1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $v$ , $ v $ , $  v  $ , v).	+	HS.MP.4. Model with mathematics.	
Ingonometry 7.4			
rising the coordinates of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. Trigonometry 7.4	+	and quantitatively.	
HS.N-VM.3. Solve problems involving velocity and other quantities that can be represented by vectors. Connections: 11-12.RST.9 SCHS-S5C2-01; SCHS-S5C2-02; SCHS-S5C2-06; 11-12.WHST.2d <b>Trigonometry 7.4</b>	+	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> </ul>	<ul> <li>Examples:</li> <li>A motorboat traveling from one shore to the other at a rate of 5 m/s east encounters a current flowing at a rate of 3.5 m/s north. <ul> <li>What is the resultant velocity?</li> <li>If the width of the river is 60 meters wide, then how much time does it take the boat to travel to the opposite shore?</li> <li>What distance downstream does the boat reach the opposite shore?</li> </ul> </li> <li>A ship sails 12 hours at a speed of 15 knots (nautical miles per hour) at a heading of 68[°] north of east. It then turns to a heading of 75[°] north of east and travels for 5 hours at 8 knots. Find its position north and east of its starting point. (For this problem, assume the earth is flat.)</li> <li>The solution may require an explanation, orally or in written form, that includes understanding of velocity and other relevant quantities</li> </ul>

Number and Quantity: Vector and Matrix Quantities (N-VM)			
Perform operations on vecto	rs.		
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.N-VM.4. Add and subtract vectors.	+	HS.MP.2. Reason abstractly and quantitatively.	Addition of vectors is used to determine the resultant of two given vectors. This can be done by lining up the vectors end to end, adding the components, or using the parallelogram rule. Students may use applets to help them visualize operations of
Trigonometry 7.4		HS.MP.4. Model with	vectors given in rectangular or polar form.
<ul> <li>Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</li> <li>Trigonometry 7.4</li> </ul>	+	mathematics. HS.MP.5. Use appropriate tools strategically.	(a, b) (c, d)
b. Given two vectors in magnitude and direction	+		
form, determine the magnitude and direction of their sum. <b>Trigonometry 7.4</b>			<ul> <li>Example:</li> <li>Given two vectors u and v, can the magnitude of the resultant be found by adding the magnitude of each vector? Use an example to illustrate your explanation.</li> </ul>

Number and Quantity: Vec	tor and I	Matrix Quantities (N-VM)	
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
c. Understand vector subtraction $v - w$ as $v +$ (-w), where $-w$ is the additive inverse of $w$ , with the same magnitude as $w$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component- wise.	+		<ul> <li>If u = (-2, -8) and v = (2, 8), find u + v, u + (-v), and u - v. Explain the relationship between u + (-v) and u - v in terms of the vector components.</li> <li>A plane is flying due east at an average speed of 500 miles per hour. There is a crosswind from the south at 60 miles per hour. What is the magnitude and direction of the resultant?</li> </ul>
Connection: ETHS-S6C1-03			
Trigonometry 7.4			
HS.N-VM.5. Multiply a vector	+	HS.MP.2. Reason abstractly	The result of multiplying a vector $v$ by a positive scalar $c$ is a vector in the same
Trigonometry 7 4		anu quantitativery.	reversed by scalar multiplication. Students will represent scalar multiplication
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component- wise, e.g., as $c(v_x, v_y) =$ $(cv_x, cv_y)$ . <b>Trigonometry 7.4</b>	+	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	graphically and component-wise. Students may use applets to help them visualize operations of vectors given in rectangular or polar form. Example: • Given $u = \langle 2, 4 \rangle$ , write the components and draw the vectors for $u$ , $2u$ , $\frac{1}{2}u$ , and $-u$ . How are the vectors related?

Number and Quantity: Vec	tor and	Matrix Quantities (N-VM)	
Perform operations on vecto	rs.		
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
b. Compute the magnitude of a scalar multiple $cv$ using $  cv   =  c v$ . Compute the direction of $cv$ knowing that when $ c v \neq 0$ , the direction of $cv$ is either along $v$ (for $c > 0$ ) or against $v$ (for $c < 0$ ).	+		
Connection: ETHS-S6C1-03 Trigonometry 7.4			

Number and Quantity: Vector and Matrix Quantities (N-VM)				
Perform operations on matri				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.N-VM.6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	9-10 +	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics.	Students may use graphing calculators and spreadsheets to create and perform operations on matrices. The adjacency matrix of a simple graph is a matrix with rows and columns labeled by graph vertices, with a 1 or a 0 in position ( $v_i$ , $v_j$ ) according to whether $v_i$ and $v_j$ are adjacency at a "1" indicates that there is a superstice between the two vertices.	
Connections: 9-10.RST.7; 9-10.WHST.2f; 11-12.RST.9; 11-12.WHST.2e ETHS-S6C2-03; College Algebra 6.5		HS.MP.5. Use appropriate tools strategically.	adjacent or not. A "1" indicates that there is a connection between the two vertices, and a "0" indicates that there is no connection. Example: • Write an inventory matrix for the following situation. A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of paint, 12 brushes, and 17 canvases. For class 2, the teacher buys 20 tubes of paint, 14 brushes and 15 canvases. Next year, she has 3 times as many students in each class. What affect does this have on the amount of supplies? Solution: Year 1 P B C Class 1 $\begin{bmatrix} 24 & 12 & 17 \\ 20 & 14 & 15 \end{bmatrix}$ Year 2 P B C Class 1 $\begin{bmatrix} 72 & 36 & 51 \\ 60 & 42 & 45 \end{bmatrix}$	
relationships in a network. Connections: 9-10.RST.7; 9-10.WHST.2f; 11-12.RST.9; 11-12.WHST.2e ETHS-S6C2-03; College Algebra 6.5		mathematics. HS.MP.5. Use appropriate tools strategically.	graph vertices, with a 1 or a 0 in position (vi, vj) according to whether vi and vj a adjacent or not. A "1" indicates that there is a connection between the two vertices and a "0" indicates that there is no connection.Example:• Write an inventory matrix for the following situation. A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of p 12 brushes, and 17 canvases. For class 2, the teacher buys 20 tubes of paint, 14 brushes and 15 canvases. Next year, she has 3 times as mar students in each class. What affect does this have on the amount of supplies?Solution:Year 1P B C Class 1 Class 220 14 15Year 2P B C Class 1 Class 2P B C Class 1 Class 2P B C Class 1 Class 2P B C Class 1 Class 2C Class 1 Class 2P B C Class 2C Class 2P B C Class 2C Class 2P B C Class 2C Class 2C Class 2P B C Class 2C Class 2C Class 2C Class 2P B C Class 2C Class 2C Class 2C Class 2C Class 2C Class 2C Class 2C Class 2 </td	

Number and Quantity: Vector and Matrix Quantities (N-VM)				
Perform operations on matrices and use matrices in applications.				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.N-VM.7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. Connections: 9-10.RST.3; ETHS-S6C2-03	9-10	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically	Students may use graphing calculators and spreadsheets to create and perform operations on matrices. Example: $-3\begin{bmatrix} -7 & 19 & 15\\ 41 & -63 & 20\\ 2 & 0 & -8 \end{bmatrix}$	
College Algebra 6.1			• The following is an inventory matrix for Company A's jellybean, lollipop, and gum flavors. The price per unit is \$0.03 for jelly beans, gum, and lollipops. Determine the gross profit for each flavor and for the entire lot. F1 F2 F3 F4 F5 F6 F7 C1 $\begin{bmatrix} 327 818 465 211 127 134 705\\ 513 222 312 446 645 671 101\\ C3 \begin{bmatrix} 878 901 51 156 711 423 344 \end{bmatrix}$ F1 = Vanilla C1 = Jelly beans C2 = Lollipops C3 = Gum F1 = Vanilla F1 = Tangerine F5 = Coconut F6 = Mint E7 = Lipaging	

Number and Quantity: Vec	tor and I	Matrix Quantities (N-VM)			
Perform operations on matric	Perform operations on matrices and use matrices in applications.				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.N-VM.8. Add, subtract, and multiply matrices of appropriate dimensions.	9-10	HS.MP.2. Reason abstractly and quantitatively.	Students may use graphing calculators and spreadsheets to create and perform operations on matrices.		
Connections: 9-10.RST.3; ETHS-S6C2-03		HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate	<ul> <li>Find 2A – B + C and A • B given Matrices A, B and C below.</li> </ul>		
College Algebra 6.1- 6.2		tools strategically.	Matrix AMatrix BMatrix C $\begin{bmatrix} -7 & 19 & 15\\ 41 & -63 & 20\\ 2 & 0 & -8 \end{bmatrix}$ $\begin{bmatrix} 23 & 18 & 55\\ -18 & -47 & 11\\ 39 & -6 & -8 \end{bmatrix}$ $\begin{bmatrix} -4 & 7 & 12\\ 51 & 9 & 80\\ 13 & 72 & 8 \end{bmatrix}$		
HS.N-VM.9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	9-10	HS.MP.2. Reason abstractly and quantitatively. HS.MP.6. Attend to precision.	Students may use graphing calculators and spreadsheets to create and perform operations on matrices. Example: • Given $A = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 6 & -2 \\ 9 & 7 \end{bmatrix}$ ;		
Connections: ETHS-S6C2-03; 9-10.WHST.1e			determine if the following statements are true: AB = BA AB = A(BC)		
College Algebra 6.2					

Number and Quantity: Vector and Matrix Quantities (N-VM)				
Perform operations on matrices and use matrices in applications.				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.N-VM.10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.6. Attend to precision.		
College Algebra 6.7				
HS.N-VM.11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. Connections: ETHS-S6C1-03; 11-12.WHST.1a College Algebra chapter 6.	+	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	A matrix is a two dimensional array with rows and columns; a vector is a one dimensional array that is either one row or one column of the matrix. Students will use matrices to transform geometric objects in the coordinate plane. Students may demonstrate transformations using dynamic geometry programs or applets. They will explain the relationship between the ordered pair representation of a vector and its graphical representation.	
HS.N-VM.12. Work with $2 \times 2$ matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. Connection: ETHS-S6C1-03	+	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Students should be able to utilize matrix multiplication to perform reflections, rotations and dilations, and find the area of a parallelogram. Students may demonstrate these relationships using dynamic geometry programs or applets.	

#### High School – Algebra Overview

#### Seeing Structure in Expressions (A-SSE)

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

#### Arithmetic with Polynomials and Rational Expressions (A-APR)

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

#### **Creating Equations (A-CED)**

Create equations that describe numbers or relationships

#### **Reasoning with Equations and Inequalities (A-REI)**

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

#### **Mathematical Practices (MP)**

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

#### High School – Algebra

**Expressions.** An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price *p*. Rewriting p + 0.05p as 1.05p shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

**Equations and Inequalities.** An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of  $x^2 - 2 = 0$  are real numbers, not rational numbers; and the solutions of  $x^2 + 2 = 0$  are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid,  $A = ((b_1+b_2)/2)h$ , can be solved for *h* using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

**Connections to Functions and Modeling**. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra: Seeing Structure in Expressions (A-SSE)						
Interpret the structure of exp	ressions					
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.A-SSE.1. Interpret expressions that represent a quantity in terms of its context.	9-10 ★	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.2. Reason abstractly and quantitatively.	Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret there meaning in terms of a context.			
Elementary Algebra 1.6		HS.MP.4. Model with mathematics.				
Intermediate Algebra 2.1		HS.MP.7. Look for and make use of structure.				
a. Interpret parts of an expression, such as terms, factors, and coefficients.	9-10 ★					
Connection: 9-10.RST.4 Elementary Algebra 2.4						

Algebra: Seeing Structure in Expressions (A-SSE)						
Interpret the structure of expressions						
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
<ul> <li>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)ⁿ as the product of P and a factor not depending on P.</li> <li>Elementary Algebra 2.4</li> </ul>	9-10 ★					
HS.A-SSE.2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .	9-10	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.	Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. Example: Factor $x^3 - 2x^2 - 35x$			
Elementary Algebra 4.2 Intermediate 3.2						

Algebra: Seeing Structure in Expressions (A-SSE)						
Write expressions in equivalent for	Write expressions in equivalent forms to solve problems					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain	9-10 +	HS.MP.1. Make sense of problems and persevere in	Students will use the properties of operations to create equivalent expressions.			
properties of the quantity represented by the expression.	(	Solving them.	Examples: $\sum_{n=1}^{\infty} \sum_{i=1}^{\infty} 2in^2 + n = (i - i) = (n - 2)(n + 4)$ in factors d			
Connections: 9-10.WHST.1c; 11-12.WHST.1c		quantitatively.	• Express $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$ in factored form and use your answer to say for what values of x the expression is zero			
Elementary Algebra 1.4,			<ul> <li>Write the expression below as a constant times a power of x</li> </ul>			
1.6, chapter 4.			and use your answer to decide whether the expression gets			
Intermediate Algebra 2.5,			larger or smaller as x gets larger.			
chapter 3.			$(2x^3)^2(3x^4)$			
College Algebra 3.4			$ \circ \frac{(r^2)^3}{(r^2)^3} $			
a. Factor a quadratic expression to reveal the zeros of the function it defines	9-10 ★	HS.MP.4. Model with mathematics.				
Elementary Algebra 4.6						
Intermediate Algebra 3.6						
College Algebra 3.4, 3.5						
<i>b.</i> Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	9-10 ★	HS.MP.7. Look for and make use of structure.				
Elementary Algebra 9.2						
Intermediate Algebra 6.6						
College Algebra 3.1						

c. Use the properties of	+	]	
exponents to transform	★		
expressions for exponential			
functions. For example the			
expression 1.15 ^t can be			
rewritten as (1.15 ^{1/12} ) ^{12t} ≈			
1.012 ^{12t} to reveal the			
approximate equivalent			
monthly interest rate if the			
annual rate is 15%.			
Intermediate Algebra 11.5			
College Algebra 4.4			

Algebra: Seeing Structure in Expressions (A-SSE)					
Standards	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. <i>For example,</i> <i>calculate mortgage</i> <i>payments.</i>	(	<ul><li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li><li>HS.MP.4. Model with mathematics.</li></ul>	<ul> <li>Example:</li> <li>In February, the Bezanson family starts saving for a trip to Australia in September. The Bezanson's expect their vacation to cost \$5375. They start with \$525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip?</li> </ul>		
Connection: 11-12.RST.4 College Algebra 7.3		HS.MP.7. Look for and make use of structure.			

Algebra: Arithmetic with Polynomials and R	Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)					
Perform arithmetic operation	s on poly	nomials.				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.A-APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	9-10	HS.MP.8. Look for regularity in repeated reasoning.				
Connection: 9-10.RST.4						
Elementary Algebra 1.2 – 1.4 Intermediate Algebra 2.2, 2.4 College Algebra 3.2						

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)			
Understand the relationship between zeros and factors of polynomials			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-APR.2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number <i>a</i> , the remainder on division by $x - a$ is $p(a)$ , so p(a) = 0 if and only if $(x - a)$ is a factor of $p(x)$ .	+	HS.MP.2. Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others.	<ul> <li>The Remainder theorem says that if a polynomial p(x) is divided by x - a, then the remainder is the constant p(a). That is, p(x)=q(x)(x-a)+p(a). So if p(a) = 0 then p(x) = q(x)(x-a).</li> <li>Let p(x)=x⁵-3x⁴+8x²-9x+30. Evaluate p(-2). What does your answer tell you about the factors of p(x)? [Answer: p(-2) = 0 so x+2 is a factor.]</li> </ul>
College Algebra 3.3			
HS.A-APR.3. Identify zeros of polynomials when suitable factorizations are available,	*	HS.MP.2. Reason abstractly and quantitatively.	Graphing calculators or programs can be used to generate graphs of polynomial functions.
and use the zeros to construct a rough graph of		HS.MP.4. Model with mathematics.	Example:
the function defined by the polynomial.		HS.MP.5. Use appropriate tools strategically.	• Factor the expression $x^3 + 4x^2 - 59x - 126$ and explain how your answer can be used to solve the equation $x^3 + 4x^2 - 59x - 126 = 0$ . Explain why
College Algebra 3.4-3.5			the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x) = x^3 + 4x^2 - 59x - 126$ .

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)			
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-APR.4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Elementary Algebra 2.6	+	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	Examples: Use the distributive law to explain why $x^2 - y^2 = (x - y)(x + y)$ for any two numbers x and y. Derive the identity $(x - y)^2 = x^2 - 2xy + y^2$ from $(x + y)^2 = x^2 + 2xy + y^2$ by replacing y by $-y$ . Use an identity to explain the pattern $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ [Answer: $(n + 1)^2 - n^2 = 2n + 1$ for any whole number <i>n</i> .]
HS.A-APR.5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) <b>College Algebra 7.4</b>	*	<ul> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> </ul>	Examples: • Use Pascal's Triangle to expand the expression $(2x-1)^4$ . • Find the middle term in the expansion of $(x^2 + 2)^{18}$ . • 1 1 1 2 1 1 2 1 1 3 3 1 $(x+1)^3 = x^3 + 3x^2 + 3x + 1$ 1 4 6 4 1 $\uparrow \uparrow $

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)			
Rewrite rational expressions			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-APR.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and r(x) are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.7. Look for and make use of structure.	<ul> <li>The polynomial q(x) is called the quotient and the polynomial r(x) is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes.</li> <li>Examples: <ul> <li>Find the quotient and remainder for the rational expression x³-xx²+x-6/x²+x²</li> <li>and use them to write the expression in a different form.</li> <li>Express f(x) = xx+1/x-1 in a form that reveals the horizontal asymptote of its graph. [Answer: f(x) = xx+1/x-1 = x(x+1)+z/x-1 = 2 + x/x-1, so the horizontal asymptote is y = 2.]</li> </ul> </li> </ul>
Elementary Algebra 2.7 Intermediate Algebra 2.5 College Algebra 3.3			

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)			
Rewrite rational expressions			
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-APR.7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	9-10	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	<ul> <li>Examples:</li> <li>Use the formula for the sum of two fractions to explain why the sum of two rational expressions is another rational expression.</li> <li>Express ¹/_{x²+1} - ¹/_{x²-1} in the form a(x)/b(x), where a(x) and b(x) are polynomials.</li> </ul>
Elementary Algebra chapter 5 Intermediate Algebra chapter 4 College Algebra 3.6			

Algebra: Creating Equations * (A-CED)			
Create equations that describe numbers or relationships			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-CED.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include</i> <i>equations arising from linear</i> <i>and quadratic functions, and</i> <i>simple rational and</i> <i>exponential functions.</i> Elementary Algebra chapter 3 Intermediate Algebra chapter 4, 6, 7, 11 College Algebra chapter 1, 4	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	<ul> <li>Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.</li> <li>Examples: <ul> <li>Given that the following trapezoid has area 54 cm², set up an equation to find the length of the base, and solve the equation.</li> <li>10 cm</li> <li>6 cm¹/₁</li> </ul> </li> <li>Lava coming from the eruption of a volcano follows a parabolic path. The height <i>h</i> in feet of a piece of lava <i>t</i> seconds after it is ejected from the volcano is given by <i>h(t) = -t² + 16t + 936</i>. After how many seconds does the lava reach its maximum height of 1000 feet?</li> </ul>
HS.A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Elementary Algebra chapter 7 Intermediate Algebra chapter 9 College Algebra chapter 5	9-10 (	<ul><li>HS.MP.2. Reason abstractly and quantitatively.</li><li>HS.MP.4. Model with mathematics.</li><li>HS.MP.5. Use appropriate tools strategically.</li></ul>	

Algebra: Creating Equations * (A-CED)			
Create equations that describe numbers or relationships			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. <b>Elementary Algebra</b> <b>3.5</b> <b>Intermediate Algebra</b> <b>8.7</b> <b>College Algebra 1.7</b>	9-10 ★	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	<ul> <li>Example:</li> <li>A club is selling hats and jackets as a fundraiser. Their budget is \$1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs \$5 and each jacket costs \$8.</li> <li>Write a system of inequalities to represent the situation.</li> <li>Graph the inequalities.</li> <li>If the club buys 150 hats and 100 jackets, will the conditions be satisfied?</li> <li>What is the maximum number of jackets they can buy and still meet the conditions?</li> </ul>
HS.A-CED.4. Rearrange	9-10	HS.MP.2. Reason	Examples:
formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. Elementary Algebra 3.3 Intermediate Algebra 6.1 College Algebra 1.2, 1.6	(	abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	<ul> <li>The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation a2 + b2 = c2.</li> <li>Why might the theorem need to be solved for c?</li> <li>Solve the equation for c and write a problem situation where this form of the equation might be useful.</li> <li>Solve V = ⁴/₃πr³ for radius r.</li> <li>Motion can be described by the formula below, where t = time elapsed, u=initial velocity, a = acceleration, and s = distance traveled s = ut+½at²</li> </ul>
			<ul> <li>Why might the equation need to be rewritten in terms of <i>a</i>?</li> <li>Rewrite the equation in terms of <i>a</i>.</li> </ul>
Algebra: Reasoning with Equations and Inequalities (A-REI)			
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Understand solving equations as a process of reasoning and explain the reasoning			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Elementary Algebra 3.1 Intermediate Algebra 6.1 College Algebra 1.2, 1.6	9-10	<ul><li>HS.MP.2. Reason abstractly and quantitatively.</li><li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li><li>HS.MP.7. Look for and make use of structure.</li></ul>	<ul> <li>Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions.</li> <li>Examples: <ul> <li>Explain why the equation x/2 + 7/3 = 5 has the same solutions as the equation 3x + 14 = 30. Does this mean that x/2 + 7/3 is equal to 3x + 14?</li> <li>Show that x = 2 and x = -3 are solutions to the equation x² + x = 6. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning.</li> </ul> </li> </ul>
HS.A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. <b>Elementary Algebra</b> <b>5.5</b> , 8.5 Intermediate Algebra <b>4.5</b> , 5.7 College Algebra 1.5	9-10	<ul><li>HS.MP.2. Reason abstractly and quantitatively.</li><li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li><li>HS.MP.7. Look for and make use of structure.</li></ul>	Examples: • $\sqrt{x+2} = 5$ • $\frac{7}{8}\sqrt{2x-5} = 21$ • $\frac{x+2}{x+2} = 2$ Solv • $\sqrt{3x-7} = -4$

Algebra: Reasoning with Equations and Inequalities (A-REI)			
Solve equations and inequalities in one variable			
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Elementary Algebra chapter 3 Intermediate Algebra chapter 6, 7 College Algebra 1.1, 1.2, 1.5	9-10	<ul> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	Examples: • $-\frac{7}{3}y - 8 = 111$ • $3x > 9$ • $ax + 7 = 12$ • $\frac{3+x}{7} = \frac{x-9}{4}$ • Solve for <i>x</i> : $2/3x + 9 < 18$
HS.A-REI.4. Solve quadratic equations in one variable. Elementary Algebra chapter 9 Intermediate Algebra 6.4 - 6.7 College Algebra 1.4, 3.1	*	<ul><li>HS.MP.2. Reason abstractly and quantitatively.</li><li>HS.MP.7. Look for and make use of structure.</li><li>HS.MP.8. Look for and express regularity in repeated reasoning.</li></ul>	Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c =$ 0 to the behavior of the graph of $y = ax^2 + bx + c$ .Value of DiscriminantNature of RootsNature of Graph $b^2 - 4ac = 0$ 1 real rootsintersects x-axis once $b^2 - 4ac > 0$ 2 real rootsintersects x-axis twice $b^2 - 4ac < 0$ 2 complex rootsdoes not intersect x-axis

Algebra: Reasoning with E	quation	s and Inequalities (A-REI)	
Solve equations and inequal	ities in or	ne variable	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
a. Use the method of completing the square to transform any quadratic equation in <i>x</i> into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Elementary Algebra 9.2 Intermediate Algebra	*		<ul> <li>Are the roots of 2x² + 5 = 2x real or complex? How many roots does it have? Find all solutions of the equation.</li> <li>What is the nature of the roots of x² + 6x + 10 = 0? Solve the equation using the quadratic formula and completing the square. How are the two methods related?</li> </ul>
6.6	ſ		
	ſ		
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers <i>a</i> and <i>b</i> . <b>Elementary Algebra</b> <b>chapter 9</b> <b>Intermediate Algebra</b> <b>6.4 - 6.7</b> <b>College Algebra 1.4,</b> <b>3.1</b>	*		

Algebra: Reasoning with Equations and Inequalities (A-REI)			
Solve systems of equations			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-REI.5. Prove that, given a system of two equations in	9-10	HS.MP.2. Reason abstractly and quantitatively.	Example:
two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		HS.MP.3. Construct viable arguments and critique the reasoning of others.	Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that they two numbers, <i>x</i> and <i>y</i> , satisfy the equations $x + y = 10$ and $x - y = 4$ .
Intermediate Algebra 9.2 College Algebra 5.1			

Algebra: Reasoning with Equations and Inequalities (A-REI)			
-			
<u>Label</u>	Mathematical Practices	Explanations and Examples	
9-10	<ul> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	<ul> <li>The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.</li> <li>Examples: <ul> <li>José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.</li> <li>Before: ?</li> <li>José</li></ul></li></ul>	
	uation: L <u>abel</u> 9-10	uations and Inequalities (A-REI)LabelMathematical Practices9-10HS.MP.2. Reason abstractly and quantitatively.HS.MP.4. Model with mathematics.HS.MP.5. Use appropriate tools strategically.HS.MP.6. Attend to precision.HS.MP.7. Look for and make use of structure.HS.MP.8. Look for and express regularity in repeated reasoning.	

Algebra: Reasoning with Equations and Inequalities (A-REI)			
Solve systems of equations			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2$ + $y^2 = 3$ . <b>College Algebra 5.4</b>	*	<ul> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	<ul> <li>Example:</li> <li>Two friends are driving to the Grand Canyon in separate cars. Suzette has been there before and knows the way but Andrea does not. During the trip Andrea gets ahead of Suzette and pulls over to wait for her. Suzette is traveling at a constant rate of 65 miles per hour. Andrea sees Suzette drive past. To catch up, Andrea accelerates at a constant rate. The distance in miles (<i>d</i>) that her car travels as a function of time in hours (<i>t</i>) since Suzette's car passed is given by <i>d</i> = 3500<i>t</i>².</li> <li>Write and solve a system of equations to determine how long it takes for Andrea to catch up with Suzette.</li> </ul>
HS.A-REI.8. Represent a system of linear equations as a single matrix equation in a vector variable. Intermediate Algebra 9.6 College Algebra chapter 6	*		Example: • Write the system $\begin{cases} -b + 2c = 4 \\ a + b - c = 0 \\ 2a + 3c = 11 \end{cases}$ Identify the coefficient matrix, the variable matrix, and the constant matrix.

Algebra: Reasoning with E	quation	s and Inequalities (A-REI)	
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-REI.9. Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). Connection: ETHS-S6C2-03 <b>College Algebra 6.7</b>	*	HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	Students will perform multiplication, addition, subtraction, and scalar multiplication of matrices. They will use the inverse of a matrix to solve a matrix equation. Students may use graphing calculators, programs, or applets to model and find solutions for systems of equations. Example: • Solve the system of equations by converting to a matrix equation and using the inverse of the coefficient matrix. $\begin{cases} 5x+2y=4\\ 3x+2y=0 \end{cases}$ Solution: Matrix $A = \begin{bmatrix} 5 & 2\\ 3 & 2 \end{bmatrix}$ Matrix $X = \begin{bmatrix} x\\ y \end{bmatrix}$ Matrix $A = \begin{bmatrix} 4\\ 0 \end{bmatrix}$ $Matrix A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2}\\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}$ $X = A^{-1}B$ $\begin{bmatrix} x\\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2}\\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix} \begin{bmatrix} 4\\ 0 \end{bmatrix} = \begin{bmatrix} 2\\ -3 \end{bmatrix}$

Algebra: Reasoning with Equations and Inequalities (A-REI)				
Represent and solve equation	Represent and solve equations and inequalities graphically			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). <b>Elementary Algebra</b> <b>chapter 6</b> <b>Intermediate Algebra</b> <b>chapter 8</b> <b>College Algebra</b> <b>chapter 8</b>	9-10	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics.	<ul> <li>Which of the following points is on the circle with equation (x - 1)² + (y + 2)² = 5?</li> <li>(a) (1, -2) (b) (2, 2) (c) (3, -1) (d) (3, 4)</li> </ul>	

Algebra: Reasoning with Equations and Inequalities (A-REI)				
Represent and solve equation	Represent and solve equations and inequalities graphically			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.A-REI.11. Explain why the <i>x</i> -coordinates of the points where the graphs of the equations $y = f(x)$ and $y =$ g(x) intersect are the solutions of the equation $f(x)$ = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Connection: ETHS-S6C2-03 Intermediate Algebra chapter 9 College Algebra	+ *	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions. Example: • Given the following equations determine the <i>x</i> value that results in an equal output for both functions. f(x) = 3x - 2 $g(x) = (x + 3)^2 - 1$	
College Algebra chapter 5				

Algebra: Reasoning with Equations and Inequalities (A-REI) Represent and solve equations and inequalities graphically			
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.A-REI.12. Graph the solutions to a linear inequality in two variables as a half- plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes. Elementary Algebra 3.4, 6.5, 7.4 Intermediate Algebra 7.1, 8.6, 8.7 College Algebra 1.6	9-10	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities. Examples: • Graph the solution: $y \le 2x + 3$ . • A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines, but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set. • Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system. $\begin{cases} x-3y > 0 \\ x+y \le 2 \\ x+3y > -3 \end{cases}$ Solution: (3, 2) is not an element of the solution set (graphically or by substitution).

#### **High School - Functions Overview**

#### Interpreting Functions (F-IF)

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

#### **Building Functions (F-BF)**

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

#### Linear, Quadratic, and Exponential Models (F-LE)

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

#### **Trigonometric Functions (F-TF)**

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

#### **Mathematical Practices (MP)**

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

#### **High School - Functions**

Functions describe situations where one quantity determines another. For example, the return on \$10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule T(v) = 100/v expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like f(x) = a + bx; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

#### Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions:	-		
Interpreting Functions (F-I	F) function	and use function notation	
Standards		Mathematical Practices	Explanations and Examples
Students are expected to:	Laver	Mathematical Flactices	
HS.F-IF.1. Understand that a function from one set (called the domain) to another set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <i>f</i> is a function and <i>x</i> is an element of its domain, then $f(x)$ denotes the output of <i>f</i> corresponding to the input <i>x</i> . The graph of <i>f</i> is the graph of the equation $y = f(x)$ .	9-10	HS.MP.2. Reason abstractly and quantitatively.	The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain.
10.1			
College Algebra 2.3			
HS.F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	9-10	HS.MP.2. Reason abstractly and quantitatively.	The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain. Examples: • If $f(x) = x^2 + 4x - 12$ , find $f(2)$ .
Connection: 9-10.RST.4			<ul> <li>Let f(x) = 2(x+3)². Find f(3), f(-1/2), f(a), and f(a - h)</li> <li>If P(t) is the population of Tucson t years after 2000, interpret the statements</li> </ul>
Intermediate Algebra 10.3			P(0) = 487,000 and $P(10)-P(9) = 5,900$ .
College Algebra 2.4			

Functions: Interpreting Functions (F-IF)			
Standards	Standards I abel Mathematical Practices Explanations and Examples		
Students are expected to:			
HS.F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$ , $f(n+1) = f(n) + f(n-1)$ for $n \ge 1$ .	9-10	HS.MP.8. Look for and express regularity in repeated reasoning.	
College Algebra 7.4			

Functions: Interpreting Functions (F-IF)			
Interpret functions that arise in applications in terms of the context			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features</i> <i>include: intercepts; intervals</i> <i>where the function is</i> <i>increasing, decreasing,</i> <i>positive, or negative; relative</i> <i>maximums and minimums;</i> <i>symmetries; end behavior;</i> <i>and periodicity.</i>	*	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.	Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology. Examples: • A rocket is launched from 180 feet above the ground at time $t = 0$ . The function that models this situation is given by $h = -16t^2 + 96t + 180$ , where $t$ is measured in seconds and $h$ is height above the ground measured in feet. • What is a reasonable domain restriction for $t$ in this context? • Determine the height of the rocket two seconds after it was launched. • Determine the maximum height obtained by the rocket. • Determine the time when the rocket is 100 feet above the ground. • Determine the time at which the rocket hits the ground. • Determine the time at which the rocket hits the ground. • How would you refine your answer to the first question based on your response to the second and fifth questions? • Compare the graphs of $y = 3x^2$ and $y = 3x^3$ .
Connections: ETHS-S6C2.03; 9-10.RST.7; 11-12.RST.7 Intermediate 8.5 College Algebra 2.5, 3.1, 4.1, 4.2			<ul> <li>Let R(x) = 2/√x-2. Find the domain of R(x). Also find the range, zeros, and asymptotes of R(x).</li> <li>Let f(x) = 5x³ - x² - 5x + 1. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.</li> <li>It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.</li> </ul>

Functions: Interpreting Functions (F-IF)			
Interpret functions that arise in applications in terms of the context			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	9-10 ★	HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics. HS.MP.6. Attend to precision.	Students may explain orally, or in written format, the existing relationships.
Connection: 9-10.WHST.2f			
Intermediate Algebra 10.2 College Algebra 2.5			

Functions: Interpreting Functions (F-IF)			
Interpret functions that arise in applications in terms of the context			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.F-IF.6. Calculate and	9-10	HS.MP.2. Reason abstractly	The average rate of change of a function $y = f(x)$ over an interval [a,b] is
interpret the average rate of	*	and quantitatively.	Ay = f(b) - f(a) . In addition to finding average rates of change
change of a function		LIC MD 4 Madel with	$\frac{-a}{2x} = \frac{-a}{2}$
(presented symbolically or as		HS.MP.4. Model with	from functions given symbolically, graphically, or in a table, Students may collect
interval Estimate the rate of		mainematics.	data from experiments or simulations (ex. failing ball, velocity of a car, etc.) and find
change from a graph.		HS.MP.5. Use appropriate	average rates of change for the function modeling the situation.
5		tools strategically.	Examples:
Connections:			• Use the following table to find the average rate of change of <i>q</i> over the
ETHS-S1C2-01;			intervals [-2, -1] and [0,2]:
9-10.RST.3			
College Algebra 0.6			x  g(x)
College Algebra 2.6			-2 2
			2 -10
			• The table below shows the elapsed time when two different cars pass a 10
			20, 30, 40 and 50 meter mark on a test track.
			• For car 1, what is the average velocity (change in distance divided
			by change in time) between the 0 and 10 meter mark? Between the
			0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze
			the data to describe the motion of car 1.
			• How does the velocity of car's compare to that of car 2?
			Car 1 Car 2
			d t t
			10 4.472 1.742
			20 6.325 2.899
			30 7.746 3.831
			40 8.944 4.633
			50 10 5.348

Functions: Interpreting Functions (F-IF)			
Analyze functions using different representations			
Standards <u>Label</u> <u>Mathematical Practices</u> <u>Explanations and Examples</u>			
Students are expected to:			
HS.F-IF.7. Graph functions $\clubsuit$ HS.MP.5. Use appropriate Key characteristics include but are not limited to maxima, minima, intercepts,			
expressed symbolically and + tools strategically. symmetry, end behavior, and asymptotes. Students may use graphing calculation of the symmetry of the symmetr	tors or		
show key features of the programs, spreadsheets, or computer algebra systems to graph functions.			
graph, by hand in simple HS.MP.6. Allend to precision.			
for more complicated cases			
• Describe key characteristics of the graph of $f(x) =  x  + 5$			
Intermediate Algebra			
• Sketch the graph and identify the key characteristics of the function			
College Algebra 2.4. described below.			
$25$ $(x+2 \text{ for } x \ge 0)$			
a Graph linear and $F(x) = \begin{cases} 2c \\ 2c \\ 1 \end{cases}$			
guadratic functions and $\downarrow$			
show intercepts, maxima,			
and minima.			
ETHS-56C2-03			
6 2			
Intermediate Algebra			
• Graph the function $f(x) = 2^x$ by creating a table of values. Identify the	ΩV.		
College Algebra 2.5	Cy		
• Graph $f(x) = 2 \tan x - 1$ . Describe its domain, range, intercepts, and			

Functions: Interpreting Functions (F-IF)			
Analyze functions using different representations			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
b. Graph square root, cube	*		asymptotes.
root, and piecewise-	+		• Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$ . What are the similarities and
defined functions,	(		differences between the two graphs?
and absolute value			
functions.			
Connections:			
ETHS-S6C1-03;			
ETHS-5602-03			
8.2, 8.3			
College Algebra 2.5,			
2.6		-	
c. Graph polynomial	(		
functions, identifying	(		
zeros when suitable			
factorizations are			
available, and showing			
end behavior.			
Connections:			
ETHS-S6C1-03;			
ETHS-S6C2-03			
College Algebra 3.2			

Functions: Interpreting Fu	nctions	(F-IF)	
Analyze functions using diffe	erent repr	resentations	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
d. Graph rational	(		
functions, identifying	(		
zeros and asymptotes			
when suitable			
factorizations are			
available, and showing			
end behavior.			
Connections:			
ETHS-S6C1-03;			
ETHS-S6C2-03			
College Algebra 3.6			

Functions: Interpreting Functions (F-IF)			
Analyze functions using different representations			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
e. Graph exponential and	*		
logarithmic functions,	*		
showing intercepts and			
end behavior, and			
trigonometric			
functions, showing			
period, midline, and			
amplitude.			
_			
Connections:			
ETHS-S6C1-03;			
ETHS-S6C2-03			
Intermediate Algebra 11.1,			
11.3			
Intermediate Algebra			
chapter 11			
College Algebra			
chapter 4			
Trigonometry			
4.1-4.3			
_			

Functions: Interpreting Functions (F-IF)			
Analyze functions using different representations			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.F-IF.8. Write a function	*	HS.MP.2. Reason abstractly	
defined by an expression in		and quantitatively.	
different but equivalent forms			
to reveal and explain different		HS.MP.7. Look for and make	
properties of the function.		use of structure.	
Connection: 11 10 DOT 7			
Connection: 11-12.RS1.7			
Intermediate Algebra			
chapter 10, 11			
College Algebra			
chapters 2, 3, 4			
a. Use the process of	*		
factoring and completing			
the square in a quadratic			
function to show zeros,			
extreme values, and			
symmetry of the graph,			
terme of a context			
terms of a context.			
Connection: 11-12.RST.7			
College Algebra 3.1			

Functions: Interpreting Functions (F-IF)			
Analyze functions using different representations			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y =$ $(1.02)^t$ , $y = (0.97)^t$ , $y =$ $(1.01)^{12t}$ , $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.	*		
Connection: 11-12.RST.7 Intermediate Algebra 11.6			
College Algebra 4.4, 4.5			

Functions: Interpreting Functions (F-IF)			
Analyze functions using different representations			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Connections:	9-10	HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	Example: • Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$
ETHS-S6C1-03; ETHS-S6C2-03; 9-10.RST.7 Intermediate Algebra 8.5 College Algebra 2.5, 3.1			$\begin{array}{c c} \hline & -6 & -3 & 0 \\ \hline & -6 & -3 & 0 \\ \hline & -5 & -5 \\ \hline & -10 & -10 \\ \hline & -15 & -20 \\ \hline & & -20 \\ \hline \end{array}$

Functions: Building Functions (F-BF)				
Build a function that models	Build a function that models a relationship between two quantities			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
<ul> <li>HS.F-BF.1. Write a function that describes a relationship between two quantities.</li> <li>Connections: ETHS-S6C1-03; ETHS-S6C2-03</li> <li>Intermediate Algebra 8.4, College Algebra 2.1, 4.1, 4.2, 4.5</li> <li>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</li> <li>Connections: ETHS-S6C1-03; ETHS-S6C2-03; 9-10.RST.7; 11-12.RST.7</li> <li>College Algebra 2.1, 2.4, 2.5, 2.7, 3.1, 4.1, 4.2, 8.2</li> </ul>	9-10	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	<ul> <li>Students will analyze a given problem to determine the function expressed by identifying patterns in the function's rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function's description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.</li> <li>Examples: <ul> <li>You buy a \$10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of \$250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation.</li> <li>A cup of coffee is initially at a temperature of 93° F. The difference between its temperature and the room temperature of 68° F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.</li> <li>The radius of a circular oil slick after <i>t</i> hours is given in feet by <i>r</i> = 10t² - 0.5t, for 0 ≤ t ≤ 10. Find the area of the oil slick as a function of time.</li> </ul> </li> </ul>	

Functions: Building Functions (F-BF)			
Build a function that models a relationship between two quantities			
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
b. Combine standard	+		
function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	*		
Connections: ETHS-S6C1-03; ETHS-S6C2-03 Intermediate Algebra 10.3 College Algebra 2.4			

Functions: Building Functions (F-BF)				
Build a function that models	a relatior	nship between two quantities		
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.	↔ + ★			
Connections: ETHS-S6C1-03; ETHS-S6C2-03 Intermediate Algebra 10.3 College Algebra 2.4				
HS.F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. <b>College Algebra 7.2</b>	9-10 ★	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.8. Look for and express regularity in repeated reasoning.	<ul> <li>An explicit rule for the <i>n</i>th term of a sequence gives a_n as an expression in the term's position <i>n</i>; a recursive rule gives the first term of a sequence, and a recursive equation relates a_n to the preceding term(s). Both methods of presenting a sequence describe a_n as a function of <i>n</i>.</li> <li>Examples: <ul> <li>Generate the 5th-11th terms of a sequence if A₁= 2 and A_(n+1) = (A_n)² - 1</li> <li>Use the formula: A_n= A₁ + d(n - 1) where d is the common difference to generate a sequence whose first three terms are: -7, -4, and -1.</li> <li>There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of the year. Find the population in five years. Also, find the long-term population.</li> <li>Given the formula A_n= 2n - 1, find the 17th term of the sequence. What is the 9th term in the sequence 3, 5, 7, 9,?</li> </ul> </li> </ul>	

Functions: Building Functions (F-BF)					
Standards	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:	20001				
HS.F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and f(x + k) for specific values of $k(both positive and negative);find the value of k given thegraphs. Experiment withcases and illustrate anexplanation of the effects onthe graph using technology.Include recognizing even andodd functions from theirgraphs and algebraicexpressions for them.Connections:ETHS-S6C2-03;11-12.WHST.2eIntermediate Algebra8.5College Algebra 2.5,2.6$	+	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. Examples: • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$ , and explain the differences in terms of the algebraic expressions for the functions • $y = x^2$ • Describe effect of varying the parameters <i>a</i> , <i>h</i> , and <i>k</i> have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$ . Continued on next page		

LabelMathematical PracticesExplanations and ExamplesStudents are expected to:Intermediate Algebra 8.5 College Algebra 2.5, 2.6 Trigonometry 4.1- 4.3• Compare the shape and position of the graphs of $f(x) = e^{x}$ to $g(x) = e^{x-6} + 5$ , and explain the differences, orally or in written format, in terms of the algebraic $\frac{12}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1$	Functions: Building Functions (F-BF) Build new functions from existing functions				
Students are expected to:Intermediate Algebra 8.5 College Algebra 2.5, 2.6 	<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
<ul> <li>Compare the shape and position of the graphs of f(x) = e^x to g(x) = e^{x-6} + 5, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions</li> <li>College Algebra 2.5, 2.6</li> <li>Trigonometry 4.1- 4.3</li> <li>Describe the effect of varying the parameters a, h, and k on the shape and position of the graph f(x) = ab^{x+6} + 5 + 5 + 4 + 6 + 8 + 8 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100</li></ul>	Students are expected to:				
	Students are expected to: Intermediate Algebra 8.5 College Algebra 2.5, 2.6 Trigonometry 4.1- 4.3			<ul> <li>Compare the shape and position of the graphs of f(x) = e^x to g(x) = e^{x-6} + 5, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions</li> <li>12/10/8/22/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2/2</li></ul>	

Functions: Building Functions (F-BF)				
Build new functions from existing functions				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.F-BF.4. Find inverse functions.	+	HS.MP.2. Reason abstractly and quantitatively.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.	
Connection: ETHS-S6C2-03 Intermediate Algebra 10.4 College Algebra 2.7 Trigonometry 4.6		HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	<ul> <li>Examples:</li> <li>For the function h(x) = (x - 2)³, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist.</li> <li>Graph h(x) and h⁻¹(x) and explain how they relate to each other graphically.</li> <li>Find a domain for f(x) = 3x² + 12x - 8 on which it has an inverse. Explain why it is necessary to restrict the domain of the function</li> </ul>	
a. Solve an equation of the form $f(x) = c$ for a simple function <i>f</i> that has an inverse and write an expression for the inverse. For example, $f(x)$ =2 $x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$ .	+	HS.MP.7. Look for and make use of structure.		
Intermediate Algebra				
10.4				
College Algebra 2.7				
Trigonometry 4.6				
<ul> <li>b. Verify by composition that one function is the inverse of another.</li> <li>Intermediate Algebra 10.4</li> <li>College Algebra 2.7</li> </ul>	+			

Functions: Building Functions (F-BF)				
Build new functions from existing functions				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
<ul> <li>c. Read values of an inverse function from a graph or a table, given that the function has an inverse.</li> <li>Intermediate Algebra 10.4</li> <li>College Algebra 2.7</li> </ul>	+			
<ul> <li>d. Produce an invertible function from a non-invertible function by restricting the domain.</li> <li>Intermediate Algebra 10.4</li> <li>College Algebra 2.7</li> </ul>	+			
HS.F-BF.5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. Connection: ETHS-S6C2-03 Intermediate Algebra 11.1, 11.3 College Algebra 4.1, 4.2	+	HS.MP.2. Reason abstractly and quantitatively. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to solve problems involving logarithms and exponents. Example: • Find the inverse of $f(x) = 3(10)^{2x}$ .	

Functions: Linear, Quadratic, and Exponential Models( (F-LE)				
Construct and compare linear	r, quadrat	tic, and exponential models an	d solve problems	
Standards	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.F-LE.1. Distinguish	*	HS.MP.3. Construct viable	Students may use graphing calculators or programs, spreadsheets, or computer	
between situations that can	+	arguments and critique the	algebra systems to model and compare linear and exponential functions.	
be modeled with linear	*	reasoning of others.	Examples:	
exponential functions.		HS.MP.4. Model with mathematics.	<ul> <li>A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is</li> </ul>	
Connections:		HS.MP.5. Use appropriate	it beneficial to enroll in Plan 1? Plan 2? Plan 3?	
ETHS-S6C2-03;		tools strategically.	1. \$59.95/month for 700 minutes and \$0.25 for each additional minute,	
SSHS-S5C5-03			2. \$39.95/month for 400 minutes and \$0.15 for each additional minute,	
Intermediate Algebra		HS.MP.7. Look for and make use of structure.	<ol> <li>\$89.95/month for 1,400 minutes and \$0.05 for each additional minute</li> </ol>	
8.2 & 11.1		HS MD 9 Look for and	<ul> <li>A computer store sells about 200 computers at the price of \$1,000 per</li> </ul>	
College Algebra 4.1		express regularity in repeated	computer. For each \$50 increase in price, about ten fewer computers are	
a. Prove that linear	*	reasoning.	sold. How much should the computer store charge per computer in order to	
functions grow by equal	+	0	maximize their profit?	
differences over equal	*			
exponential functions			Students can investigate functions and graphs modeling different situations involving	
grow by equal factors			simple and compound interest. Students can compare interest rates with different	
over equal intervals.			periods of compounding (monthly, daily) and compare them with the corresponding	
			annual percentage rate. Spreadsheets and applets can be used to explore and	
Connection:			Students can use graphing calculators or programs, spreadsheets, or computer	
11-12.WHS1.1a-1e			algebra systems to construct linear and exponential functions.	
			Continued on next page	
College Algebra 4.1	•		· · · · · · · · · · · · · · · · · · ·	
D. Recognize situations in which one quantity	**		<ul> <li>A couple wants to buy a house in tive years. They need to save a down</li> </ul>	
changes at a constant	+		payment of \$6,000. They deposit \$1,000 in a bank account earning 3.25%	
rate per unit interval	*		month in order to meet their goal?	
relative to another.			<ul> <li>Sketch and analyze the graphs of the following two situations. What</li> </ul>	
			information can you conclude about the types of growth each type of interest	
Connection: 11-12.RST.4			has?	
College Algebra 1.6			$_{\odot}$ Lee borrows \$9,000 from his mother to buy a car. His mom charges him	

Functions: Linear, Quadratic, and Exponential Models( (F-LE)				
Construct and compare linear, quadratic, and exponential models and solve problems				
Standards	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
<ul> <li>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</li> <li>Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.4</li> <li>College Algebra 4.5</li> </ul>			<ul> <li>5% interest a year, but she does not compound the interest.</li> <li>Lee borrows \$9,000 from a bank to buy a car. The bank charges 5% interest compounded annually.</li> <li>Calculate the future value of a given amount of money, with and without technology.</li> <li>Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology.</li> </ul>	
HS.F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input- output pairs (include reading these from a table). Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.4; SSHS-S5C5-03 <b>College Algebra 7.1-7.3</b>		HS.MP.4. Model with mathematics. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.         Examples:         • Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the key characteristics of the graph.	

Functions: Linear, Quadratic, and Exponential Models( (F-LE)				
Construct and compare linear, quadratic, and exponential models and solve problems				
Standards	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Intermediate Algebra 11.1 College Algebra 2.5, 4.1 Discrete Structures 4.4	*	HS.MP.2. Reason abstractly and quantitatively.	Example: Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$ .	

Functions: Linear, Quadratic, and Exponential Models( (F-LE)					
Construct and compare linear, quadratic, and exponential models and solve problems					
Standards	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:					
Students are expected to: HS.F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct}$ = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.3 Intermediate Algebra 11.6 College Algebra 4.4, 4.5	+ *	HS.MP.7. Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms.Example: 		
			<i>t</i> ≈ 20.3		

Functions: Linear, Quadratic, and Exponential Models ★ (F-LE)				
Interpret expressions for fun	ctions in	terms of the situation they mo	odel	
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context. Connections: ETHS-S6C1-03; ETHS-S6C2-03; SSHS-S5C5-03; 11-12.WHST.2e		HS.MP.2. Reason abstractly and quantitatively. HS.MP.4. Model with mathematics.	<ul> <li>Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.</li> <li>Example: <ul> <li>A function of the form f(n) = P(1 + r)ⁿ is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where <i>n</i> is the number of years since the initial deposit. What is the value of <i>r</i>? What is the meaning of the constant <i>P</i> in terms of the savings account? Explain either orally or in written format.</li> </ul> </li> </ul>	
College Algebra 4.5				
Functions: Trigonometric Functions (F-TF)				
--------------------------------------------------------------------	-------	----------------------------	--------------------------------------------------------------------------------------	
Extend the domain of trigonometric functions using the unit circle				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.F-TF.1. Understand	+			
radian measure of an angle				
as the length of the arc on the				
unit circle subtended by the				
angle.				
Trigonometry 3.2				
HS.F-TF.2. Explain how the	+	HS.MP.2. Reason abstractly	Students may use applets and animations to explore the unit circle and trigonometric	
unit circle in the coordinate		and quantitatively.	functions. Students may explain (orally or in written format) their understanding.	
plane enables the extension				
of trigonometric functions to				
all real numbers, interpreted				
as radian measures of angles				
Inaversed counterclockwise				
around the unit circle.				
Connections:				
ETHS-S1C2-01:				
11-12.WHST.2b;				
11-12.WHST.2e				
Trigonometry 1.4, 3.3				

Functions: Trigonometric Functions (F-TF)				
Extend the domain of trigono	ometric fu	unctions using the unit circle		
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.F-TF.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi$ /3, $\pi$ /4 and $\pi$ /6, and use the unit circle to express the values of sine, cosine, and tangent for $\pi$ - <i>x</i> , $\pi$ + <i>x</i> , and $2\pi$ - <i>x</i> in terms of their values for <i>x</i> , where <i>x</i> is any real number. Connection: 11-12.WHST.2b <b>Trigonometry 1.2</b>	+	HS.MP.2. Reason abstractly and quantitatively. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	<ul> <li>Examples:</li> <li>Evaluate all six trigonometric functions of θ = π/3.</li> <li>Evaluate all six trigonometric functions of θ = 225°.</li> <li>Find the value of x in the given triangle where AD ⊥ DC and AC ⊥ DB m∠A = 60°, m∠C = 30°. Explain your process for solving the problem including the use of trigonometric ratios as appropriate.</li> <li>A 3 B 9 C</li> <li>Find the measure of the missing segment in the given triangle where AD ⊥ DC, AC ⊥ DB, m∠A=60°, m∠C=30°, AC = 12, AB = 3. Explain (orally or in written format) your process for solving the problem including use of trigonometric ratios as appropriate.</li> </ul>	

Functions: Trigonometric Functions (F-TF)			
Extend the domain of trigono	ometric fu	unctions using the unit circle	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.F-TF.4. Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	+	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or written format) their understanding of symmetry and periodicity of trigonometric functions.
Connections: ETHS-S1C2-01; 11-12.WHST.2c <b>Trigonometry 1.4, 3.3</b>			

Functions: Trigonometric Functions (F-TF)				
Model periodic phenomena with trigonometric functions				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. Connection: ETHS-S1C2-01 <b>Trigonometry 4.2</b>	+ ★	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	<ul> <li>Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and periodic phenomena.</li> <li>Example: <ul> <li>The temperature of a chemical reaction oscillates between a low of 20° C and a high of 120° C. The temperature is at its lowest point when <i>t</i> = 0 and completes one cycle over a six hour period.</li> <li>a) Sketch the temperature, <i>T</i>, against the elapsed time, <i>t</i>, over a 12</li> </ul> </li> </ul>	
			<ul> <li>a) Oncern the temperature, 7, against the etapsed time, 1, over a 12 hour period.</li> <li>b) Find the period, amplitude, and the midline of the graph you drew in part a).</li> <li>c) Write a function to represent the relationship between time and temperature.</li> <li>d) What will the temperature of the reaction be 14 hours after it began?</li> <li>e) At what point during a 24 hour day will the reaction have a temperature of 60° C?</li> </ul>	
HS.F-TF.6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. Connections: ETHS-S1C2-01; 11-12.WHST.2e <b>Trigonometry 4.6</b>	+		<ul> <li>Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions.</li> <li>Examples: <ul> <li>Identify a domain for the sine function that would permit an inverse function to be constructed.</li> <li>Describe the behavior of the graph of the sine function over this interval.</li> <li>Explain (orally or in written format) why the domain cannot be expanded any further.</li> </ul> </li> </ul>	

Functions: Trigonometric Functions (F-TF)					
Model periodic phenomena	Model periodic phenomena with trigonometric functions				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.F-TF.7. Use inverse functions to solve trigonometric equations that	+ ★	HS.MP.2. Reason abstractly and quantitatively.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and solve trigonometric equations.		
arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.		HS.MP.5. Use appropriate tools strategically.	<ul> <li>Example:</li> <li>Two physics students set up an experiment with a spring. In their experiment, a weighted ball attached to the bottom of the spring was pulled downward 6 inches from the rest position. It rose to 6 inches above the rest position and returned to 6 inches below the rest position once every 6</li> </ul>		
Connections: ETHS-S1C2-01; 11-12.WHST.1a <b>Trigonometry 2.2, 4.6</b>			seconds. The equation $h = -6\cos\left(\frac{\pi}{2}t\right)$ accurately models the height above and below the rest position every 6 seconds. Students may explain, orally or in written format, when the weighted ball first will be at a height of 3 inches, 4 inches, and 5 inches above rest position.		

Functions: Trigonometric	Function	s (F-TF)			
Prove and apply trigonometr	Prove and apply trigonometric identities				
Standards	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it find $\sin(\theta), \cos(\theta), \text{ or } \tan(\theta)$ given $\sin(\theta), \cos(\theta), \text{ or } \tan(\theta)$ and the quadrant of the angle.	+	HS.MP.3. Construct viable arguments and critique the reasoning of others.			
Connection: 11-12.WHST.1a-1e <b>Trigonometry 5.1</b>					
HS.F-TF.9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	+	HS.MP.3. Construct viable arguments and critique the reasoning of others.			
Connection: 11-12.WHST.1a-1e <b>Trigonometry 5.2</b>					

#### **High School - Modeling**

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO₂ over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

**Modeling Standards** Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

#### High School – Geometry Overview

#### Congruence (G-CO)

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

#### Similarity, Right Triangles, and Trigonometry (G-SRT)

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

#### Circles (G-C)

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

#### Expressing Geometric Properties with Equations (G-GPE)

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

#### Geometric Measurement and Dimension (G-GMD)

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and threedimensional objects

#### Modeling with Geometry (G-MG)

• Apply geometric concepts in modeling situations

#### **Mathematical Practices (MP)**

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

#### **High School - Geometry**

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

**Connections to Equations.** The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry: Congruence (G	Geometry: Congruence (G-CO)			
Experiment with transformat	ions in th	e plane		
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. Connection: 9-10.RST.4	9-10	HS.MP.6. Attend to precision.		
Plane Geometry 1.1, 1.6, 5.1, 8.2				

Geometry: Congruence (G-CO)				
Experiment with transformations in the plane				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Connection: ETHS-S6C1-03	9-10	HS.MP.5. Use appropriate tools strategically.	Students may use geometry software and/or manipulatives to model and compare transformations.	
Plane Geometry 6.3				
HS.G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	9-10	HS.MP.3 Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use geometry software and/or manipulatives to model transformations.	
Connections: ETHS-S6C1-03; 9-10.WHST.2c <b>Plane Geometry 6.2</b>				

Geometry: Congruence (G-CO)				
Standards	l ahel	Mathematical Practices	Explanations and Examples	
Students are expected to:	Laber	<u>mathematical Practices</u>		
HS.G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	9-10	HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	Students may use geometry software and/or manipulatives to model transformations. Students may observe patterns and develop definitions of rotations, reflections, and translations.	
Connections: ETHS-S6C1-03; 9-10.WHST.4				
Plane Geometry 6.2	0.10	LIC MD 2. Construct viable	Studente may une geometry coffuere and/or manipulatives to model transformations	
RS.G-CO.S. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	9-10	<ul><li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li><li>HS.MP.5. Use appropriate tools strategically.</li><li>HS.MP.7. Look for and make use of structure.</li></ul>	and demonstrate a sequence of transformations that will carry a given figure onto another.	
Connections: ETHS-S6C1-03; 9-10.WHST.3 <b>Plane Geometry 6.2</b>				

Geometry: Congruence (G-CO)				
Understand congruence in terms of rigid motions				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				

Geometry: Congruence (G-CO)					
Understand congruence in te	Understand congruence in terms of rigid motions				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others.	A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.		
rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.		HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	Students may use geometric software to explore the effects of rigid motion on a figure(s).		
Connections: ETHS-S1C2-01; 9-10.WHST.1e					
Plane Geometry 6.2					
HS.G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others.	A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Congruence of triangles Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.		
Connection: 9-10.WHST.1e Plane Geometry 6.2					

Geometry: Congruence (G	Geometry: Congruence (G-CO)			
Understand congruence in te	erms of ri	gid motions		
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others.		
Connection: 9-10.WHST.1e Plane Geometry 6.2				

Geometry: Congruence (G-CO)				
Prove geometric theorems				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-CO.9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about lines and angles.	
Connections: ETHS-S1C2-01; 9-10.WHST.1a-1e <b>Plane Geometry 1.6</b>				

Geometry: Congruence (Ge	-CO)		
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.G-CO.10. Prove theorems about triangles. <i>Theorems include: measures</i> <i>of interior angles of a triangle</i> <i>sum to 180°;</i> <b>Plane Geometry 4.3</b> <i>base angles of isosceles</i> <i>triangles are congruent;</i> <b>Plane Geometry 2.5</b> <i>the segment joining</i> <i>midpoints of two sides of a</i> <i>triangle is parallel to the third</i> <i>side and half the length; the</i> <i>medians of a triangle meet at</i> <i>a point.</i> <b>Plane Geometry 2.1</b> Connections: ETHS-S1C2-01:	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles.
9-10.WHST.1a-1e			
HS.G-CO.11. Prove theorems about parallelograms. <i>Theorems</i> <i>include: opposite sides are</i> <i>congruent, opposite angles</i> <i>are congruent, the diagonals</i> <i>of a parallelogram bisect</i> <i>each other, and conversely,</i> <i>rectangles are parallelograms</i> <i>with congruent diagonals.</i>	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms.
Connection: 9-10.WHST.1a-1e <b>Plane Geometry 4.2</b>			

Geometry: Congruence (G-CO) Make geometric constructions				
Standards	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Connection: ETHS-S6C1-03 <b>Plane Geometry 6.1</b>	9-10	HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.	<ul> <li>Students may use geometric software to make geometric constructions.</li> <li>Examples: <ul> <li>Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.</li> <li>Construct the circumcenter of a given triangle.</li> </ul> </li> </ul>	
HS.G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Connection: ETHS-S6C1-03	9-10	HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision.	Students may use geometric software to make geometric constructions.	

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)				
Understand similarity in terms of similarity transformations				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor:	9-10	HS.MP.2. Reason abstractly and quantitatively. HS.MP.5. Use appropriate tools strategically.	A dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations.	
Connections: ETHS-S1C2-01; 9-10.WHST.1b; 9-10.WHST.1e				
Plane Geometry 6.3				
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	9-10			
Plane Geometry 6.3				
<ul> <li>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</li> <li>Plane Geometry 6.3</li> </ul>	9-10			

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)					
Understand similarity in term	Understand similarity in terms of similarity transformations				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.G-SRT.2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate	A similarity transformation is a rigid motion followed by a dilation. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.		
similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		tools strategically. HS.MP.7. Look for and make use of structure.			
Connections: ETHS-S1C2-01; 9-10.RST.4; 9-10.WHST.1c Plane Geometry 6.3					
HS.G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others.			
Connections: ETHS-S1C2-01; 9-10.RST.7					
Plane Geometry 6.3					

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)					
Prove theorems involving sir	Prove theorems involving similarity				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.G-SRT.4. Prove theorems about triangles. <i>Theorems</i> <i>include: a line parallel to one</i> <i>side of a triangle divides the</i> <i>other two proportionally, and</i> <i>conversely;</i> <b>Plane Geometry 2.2</b> <i>the Pythagorean Theorem</i> <i>proved using triangle</i> <i>similarity.</i> <b>Plane Geometry 2.4</b> Connections: ETHS-S1C2-01; 9-10.WHST.1a-1e	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.		
HS.G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Connections: ETHS-S1C2-01; 9-10.WHST.1a-1e	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Similarity postulates include SSS, SAS, and AA. Congruence postulates include SSS, SAS, ASA, AAS, and H-L. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.		
Plane Geometry 2.3					

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)					
Define trigonometric ratios a	Define trigonometric ratios and solve problems involving right triangles				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions	9-10	HS.MP.6. Attend to precision. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may use applets to explore the range of values of the trigonometric ratios as $\theta$ ranges from 0 to 90 degrees.		
of trigonometric ratios for acute angles.			D Opposite of 0		
Connection: ETHS-S6C1-03			Adjacent to $ heta$		
Plane Geometry 3.2			sine of $\theta = \sin \theta = \frac{opposite}{hypotenuse}$ cosecant of $\theta = \csc \theta = \frac{hypotenuse}{opposite}$ cosine of $\theta = \cos \theta = \frac{adjacent}{hypotenuse}$ secant of $\theta = \sec \theta = \frac{hypotenuse}{adjacent}$ tangent of $\theta = \tan \theta = \frac{opposite}{adjacent}$ cotangent of $\theta = \cot \theta = \frac{adjacent}{opposite}$		
HS.G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles.	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others.	Geometric simulation software, applets, and graphing calculators can be used to explore the relationship between sine and cosine.		
ETHS-S1C2-01; ETHS-S6C1-03; 9-10.WHST.1c; 9-10.WHST.1e <b>Plane Geometry 3.2</b>					

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)				
Define trigonometric ratios and solve problems involving right triangles				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied	9-10 ★	HS.MP.1. Make sense of problems and persevere in solving them.	Students may use graphing calculators or programs, tables, spreadsheets, or computer algebra systems to solve right triangle problems. Example:	
problems.		HS.MP.4. Model with mathematics.	<ul> <li>Find the height of a tree to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the tree is 50 ft.</li> </ul>	
Connections: ETHS-S6C2-03; 9-10.RST.7 Plane Geometry 3.3 Trigonometry 2.4		HS.MP.5. Use appropriate tools strategically.	$\frac{28^{\circ}}{50 \text{ feet}} x$	

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)				
Apply trigonometry to general triangles				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-SRT.9. Derive the	*	HS.MP.3. Construct viable		
formula $A = \frac{1}{2} ab \sin(C)$ for	+	arguments and critique the		
the area of a triangle by		reasoning of others.		
a vertex perpendicular to the		HS MP 7 Look for and make		
opposite side.		use of structure.		
Connection: ETHS-S6C1-03				
Plane Geometry 8.2				
Trigonometry 7.3				
HS.G-SRT.10. Prove the	*	HS.MP.3. Construct viable		
Laws of Sines and Cosines	+	arguments and critique the		
and use them to solve		reasoning of others.		
problems.		HS MP 4 Model with		
Figure Geometry 3.4,3.5		mathematics		
Trigonometry 7.1,7.2		manomano.		
		HS.MP.5. Use appropriate		
11-12 WHST 1a-1e		tools strategically.		
		HS.MP.6. Attend to precision.		
		HS MP 7 Look for and make		
		HS.MF.7. LOOK IOF AND MAKE		
		HS.MP.8. Look for and		
		express regularity in repeated		
		reasoning.		

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)				
Apply trigonometry to genera	Apply trigonometry to general triangles			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-SRT.11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). Connections: 11-12.WHST.2c; 11-12.WHST.2e <b>Plane Geometry 3.5</b> <b>Trigonometry 7.1, 7.2</b>	* +	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.4. Model with mathematics.	<ul> <li>Example:</li> <li>Tara wants to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78°. From the second position, the angle between the mountain and the first position is 53°. How can Tara determine the distance of the mountain from each position, and what is the distance from each position?</li> </ul>	

Geometry: Circles (G-C)					
Understand and apply theore	Understand and apply theorems about circles				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.G-C.1. Prove that all circles are similar.	9-10	HS.MP.3. Construct viable arguments and critique the reasoning of others.	Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.		
ETHS-S1C2-01; 9-10.WHST.1a-1e Plane Geometry 6.3		HS.MP.5. Use appropriate tools strategically.			
HS.G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; <b>Plane Geometry 5.3</b> inscribed angles on a diameter are right angles; <b>Plane Geometry 5.4</b> the radius of a circle is perpendicular to the tangent where the radius intersects the circle. <b>Plane Geometry 5.3</b> Connections: 9-10.WHST.1c; 11-12.WHST.1c	9-10 +	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	<ul> <li>Examples:</li> <li>Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.</li> <li>12/10</li> <li>Find the unknown length in the picture below.</li> </ul>		

Geometry: Circles (G-C)			
Understand and apply theore	ems abou	ut circles	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.G-C.3. Construct the inscribed and circumscribed circles of a triangle, <b>Plane Geometry 6.1</b> and prove properties of angles for a quadrilateral inscribed in a circle. <b>Plane Geometry 5.1</b> Connection: ETHS-S6C1-03	+	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulation software to make geometric constructions.
HS.G-C.4. Construct a tangent line from a point outside a given circle to the circle. <b>Plane Geometry 6.1</b> Connection: ETHS-S6C1-03	+	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulation software to make geometric constructions.

Geometry: Circles (G-C)				
Find arc lengths and areas of sectors of circles				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				

Geometry: Circles (G-C)				
Find arc lengths and areas of sectors of circles				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; <b>Plane Geometry 5.4</b> derive the formula for the area of a sector. <b>Plane Geometry 8.4</b> <b>Trigonometry 3.4</b> Connections:	+	HS.MP.2 Reason abstractly and quantitatively. HS.MP.3. Construct viable arguments and critique the reasoning of others.	Students can use geometric simulation software to explore angle and radian measures and derive the formula for the area of a sector.	
ETHS-S1C2-01; 11-12.RST.4				

Geometry: Expressing Geometric Properties with Equations (G-GPE)			
I ranslate between the geom	etric des	cription and the equation for a	i conic section
<u>Standards</u>	<u>Labei</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.G-GPE.1. Derive the equation of a circle of given center and radius using the	*	HS.MP.7. Look for and make use of structure.	Students may use geometric simulation software to explore the connection between circles and the Pythagorean Theorem.
Pythagorean Theorem;		HS.MP.8. Look for and	Examples:
complete the square to find the center and radius of a circle given by an equation.		express regularity in repeated reasoning.	<ul> <li>Write an equation for a circle with a radius of 2 units and center at (1, 3).</li> <li>Write an equation for a circle given that the endpoints of the diameter are (-2, 7) and (4, -8).</li> <li>Find the center and radius of the circle 4x² + 4y² - 4x + 2y - 1 = 0.</li> </ul>
Connections: ETHS-S1C2-01; 11-12.RST.4			
Plane Geometry 11.1			
HS.G-GPE.2. Derive the equation of a parabola given a focus and directrix.	+	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and	<ul> <li>Students may use geometric simulation software to explore parabolas.</li> <li>Examples: <ul> <li>Write and graph an equation for a parabola with focus (2, 3) and directrix</li> </ul> </li> </ul>
ETHS-S1C2-01; 11-12.RST.4		reasoning.	y = 1.
Plane Geometry 11.2			
College Algebra 8.4			
HS.G-GPE.3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from	+	HS.MP.7. Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated	<ul> <li>Students may use geometric simulation software to explore conic sections.</li> <li>Example: <ul> <li>Write an equation in standard form for an ellipse with foci at (0, 5) and (2, 0) and a center at the origin.</li> </ul> </li> </ul>
the foci is constant. Connections: ETHS-S1C2-01; 11-12.RST.4 Plane Geometry 11.3, 11.4 College Algebra 8.4		reasoning.	

Explanations and Examples Omega Math: Standards and Assessment Division 9-10 = Standards for Grades 9 and 10; * = Algebra 2 standards; + = 4th credit standards, * = standards connected to mathematical modeling

Geometry: Expressing Geometric Properties with Equations (G-GPE)				
Jse coordinates to prove simple geometric theorems algebraically				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; <b>Plane Geometry 4.2</b> prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and	9-10 *	HS.MP.3 Reason abstractly and quantitatively.	<ul> <li>Students may use geometric simulation software to model figures and prove simple geometric theorems.</li> <li>Example: <ul> <li>Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.</li> </ul> </li> </ul>	
containing the point (0, 2). <b>Plane Geometry 11.1</b> Connections: ETHS-S1C2-01; 9-10.WHST.1a-1e; 11-12.WHST.1a-1e				

Geometry: Expressing Geometric Properties with Equations (G-GPE)			
Ose coordinates to prove sim	ipie geor	netric theorems algebraically	Four law attended and Four model
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.G-GPE.5. Prove the slope	9-10	HS.MP.3. Construct viable	Lines can be horizontal, vertical, or neither.
criteria for parallel and		arguments and critique the	
perpendicular lines and use		reasoning of others.	Students may use a variety of different methods to construct a parallel or
them to solve geometric			perpendicular line to a given line and calculate the slopes to compare the
problems (e.g., find the		HS.MP.8. Look for and	relationships.
equation of a line parallel or		express regularity in repeated	
perpendicular to a given line		reasoning.	
that passes through a given			
point).			
Connection:			
9-10.WHST.1a-1e			
Plane Geometry 1.7			
Elementary Algebra			
6.4			
HS G-GPE 6 Find the point	0.10	HS MP 2 Beason abstractly	Students may use geometric simulation software to model figures or line segments
on a directed line segment	3-10	and quantitatively	
between two given points that			Examples:
partitions the segment in a		HS MP 8 Look for and	
given ratio		express regularity in repeated	• Given A(3, 2) and B(6, 11)
9		reasoning	$\sim$ Find the point that divides the line segment AB two-thirds of the way
Connections:		l'ouoonnig.	from A to B
ETHS-S1C2-01			
9-10 BST 3			The point two-thirds of the way from A to B has x-coordinate two-thirds
Plane Geometry 1 1			of the way from 3 to 6 and v coordinate two-thirds of the way from 2 to
Trane Geometry 1.1			11
			So, (5, 8) is the point that is two-thirds from point A to point B.
			<ul> <li>Find the midpoint of line segment AB.</li> </ul>

Geometry: Expressing Geometric Properties with Equations (G-GPE)				
Use coordinates to prove simple geometric theorems algebraically				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-GPE.7. Use coordinates to compute perimeters of polygons <b>Plane Geometry 7.1</b> and areas of triangles <b>Plane Geometry 8.1</b> and rectangles, e.g., using the distance formula. <b>Plane Geometry 8.3</b> <b>Elementary Algebra 8.6</b> <b>Intermediate Algebra 5.6</b>	9-10 ★	<ul><li>HS.MP.2. Reason abstractly and quantitatively.</li><li>HS.MP.5. Use appropriate tools strategically.</li><li>HS.MP.6. Attend to precision.</li></ul>	Students may use geometric simulation software to model figures.	
Connections: ETHS-S1C2-01; 9-10.RST.3; 11-12.RST.3				

Geometry: Geometric Measurement and Dimension (G-GMD)				
Explain volume formulas and use them to solve problems				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-GMD.1. Give an informal argument for the formulas for the circumference of a circle, <b>Plane Geometry 7.2</b> area of a circle, <b>Plane Geometry 8.4</b> volume of a cylinder, <b>Plane Geometry 10.1</b> pyramid, and cone. <b>Plane Geometry 10.2</b> Use dissection arguments, <i>Cavalieri's principle, and</i> <i>informal limit arguments.</i> <b>Plane Geometry 10.3</b>	9-10 +	<ul><li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li><li>HS.MP.4. Model with mathematics.</li><li>HS.MP.5. Use appropriate tools strategically.</li></ul>	Cavalieri's principle is if two solids have the same height and the same cross- sectional area at every level, then they have the same volume.	
Connections: 9-10.RST.4; 9-10.WHST.1c; 9-10.WHST.1e; 11-12.RST.4; 11-12.WHST.1c; 11-12.WHST.1e				

Geometry: Geometric Measurement and Dimension (G-GMD)					
Explain volume formulas and	Explain volume formulas and use them to solve problems				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.G-GMD.2. Give an	+	HS.MP.3. Construct viable	Cavalieri's principle is if two solids have the same height and the same cross-		
informal argument using		arguments and critique the	sectional area at every level, then they have the same volume.		
Cavalieri's principle for the		reasoning of others.			
formulas for the volume of a					
sphere and other solid		HS.MP.4. Model with			
figures.		mathematics.			
Plane Geometry 10.3					
Connections: 9-10.RST.4;		HS.MP.5. Use appropriate			
9-10.WHS1.1c;		tools strategically.			
9-10.WHS1.1e;					
11-12.K51.4;					
11 12 WHST 10					
	0.10	HS MP 1 Make conce of	Missing measures can include but are not limited to slant beight, altitude, beight		
formulas for cylinders	9-10	problems and persevere in	diagonal of a prism, edge length, and radius		
pyramids cones and	×	solving them	diagonal of a prism, edge length, and radius.		
spheres to solve problems					
		HS.MP.2. Reason abstractly			
Connection: 9-10.RST.4		and guantitatively.			
		. ,			
Plane Geometry					
chapter 11					

Geometry: Geometric Measurement and Dimension (G-GMD)			
Visualize relationships between two-dimensional and three dimensional objects			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			

Geometry: Geometric Measurement and Dimension (G-GMD)			
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.G-GMD.4. Identify the shapes of two-dimensional cross-sections of three- dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. <b>Plane Geometry 6.2</b> Connection: ETHS-S1C2-01	9-10 *	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Students may use geometric simulation software to model figures and create cross sectional views. Example: • Identify the shape of the vertical, horizontal, and other cross sections of a cylinder.

Geometry: Modeling with Geometry (G-MG) ★				
Apply geometric concepts in modeling situations				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Connections: ETHS-S1C2-01; 9-10.WHST.2c	9-10 ★	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	Students may use simulation software and modeling software to explore which model best describes a set of data or situation.	
Plane Geometry 10.1				
HS.G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). Connection: ETHS-S1C2-01 <b>Plane Geometry 8.5</b> and chapter 10	9-10 ❖ ★	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	Students may use simulation software and modeling software to explore which model best describes a set of data or situation.	
### High School – Statistics and Probability Overview

### Interpreting Categorical and Quantitative Data (S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

### Making Inferences and Justifying Conclusions (S-IC)

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

### Conditional Probability and the Rules of Probability (S-CP)

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

### Using Probability to Make Decisions (S-MD)

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

### **Mathematical Practices (MP)**

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

#### High School - Statistics and Probability *

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

**Connections to Functions and Modeling.** Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) ★				
Summarize, represent, and i	nterpret o	data on a single count or mea	surement variable	
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.S-ID.1. Represent data	9-10	HS.MP.4. Model with		
with plots on the real number	*	mathematics.		
line (dot plots, histograms,				
and box plots).		HS.MP.5. Use appropriate tools strategically.		
Connections:				
SCHS-S1C1-04;				
SCHS-S1C2-03;				
SCHS-S1C2-05;				
SCHS-S1C4-02;				
SCHS-S2C1-04;				
ETHS-S6C2-03;				
9-10 BST 7				
Introduction to				
Statiation 0.1, 0.0, 0.2				
Statistics 2.1, 2.2, 2.3				
2.4				

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) *						
Summarize, represent, and interpret data on a single count or measurement variable						
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.S-ID.2. Use statistics appropriate to the shape of the data distribution to		HS.MP.2. Reason abstractly and quantitatively.	Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets.			
compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Connections: SCHS-S1C3-06; ETHS-S6C2-03; SSHS-S1C1-01 Basic Math/Pre Algebra 8.3 Introduction to Statistics 2.4, 3.5		HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics.HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	<ul> <li>Examples:</li> <li>The two data sets below depict the housing prices sold in the King River area and Toby Ranch areas of Pinal County, Arizona. Based on the prices below which price range can be expected for a home purchased in Toby Ranch? In the King River area? In Pinal County? <ul> <li>King River area? In Pinal County?</li> <li>King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}</li> <li>Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000}</li> </ul> </li> <li>Given a set of test scores: 99, 96, 94, 93, 90, 88, 86, 77, 70, 68, find the mean, median and standard deviation. Explain how the values vary about the mean and median. What information does this give the teacher?</li> </ul>			
HS.S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Connections: SSHS-S1C1-01; ETHS-S6C2-03; 9-10.WHST.1a Introduction to Statistics 2.4, 3.1, 3.2, 3.4, 3.5	9-10 ★	<ul> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.7. Look for and make use of structure.</li> </ul>	Students may use spreadsheets, graphing calculators and statistical software to statistically identify outliers and analyze data sets with and without outliers as appropriate.			

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) *					
Summarize, represent, and interpret data on a single count or measurement variable					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.S-ID.4. Use the mean and	*	HS.MP.1. Make sense of	Students may use spreadsheets, graphing calculators, statistical software and tables		
standard deviation of a data	+	problems and persevere in	to analyze the fit between a data set and normal distributions and estimate areas		
set to fit it to a normal	*	solving them.	under the curve.		
distribution and to estimate		UCMD 0 Desser shetreath	Eventilee		
population percentages.		HS.MP.2. Reason abstractly	Examples:		
sets for which such a		and quantitativery.	<ul> <li>The bar graph below gives the birth weight of a population of 100 chimpanzoos. The line shows how the weights are permally distributed.</li> </ul>		
procedure is not appropriate.		HS.MP.3. Construct viable	about the mean 3250 grams. Estimate the percent of baby chimps weighing		
Use calculators,		arguments and critique the	3000-3999 grams.		
spreadsheets, and tables to		reasoning of others.	Ŭ		
estimate areas under the		-			
normal curve.		HS.MP.4. Model with	Birth Weight Distribution for a Population		
		mathematics.			
		US MD E Llas appropriate	ş 50		
ETHS-S6C2-03		HS.MP.5. Use appropriate	± 40		
11-12 BST 7		tools strategically.	₩ 20		
11-12.RST.8:		HS.MP.6. Attend to precision.			
11-12.WRT.1b		······			
		HS.MP.7. Look for and make			
Introduction to		use of structure.			
Statistics 7.1, 7.2, 8.1,					
8.2. 8.3		HS.MP.8. Look for and	(1) (1) (1) (1) (1) (1) (1) (1) (1) (1)		
		express regularity in repeated	400 000 400 000 400 000 400 V		
		leasoning:	~>* '↓~ '↓' '5' '5' ₩* ₩*		
			Weight (grams)		
			<ul> <li>Determine which situation(s) is best modeled by a normal distribution.</li> </ul>		
			Explain your reasoning.		
			<ul> <li>Annual income of a household in the U.S.</li> </ul>		
			<ul> <li>Weight of babies born in one year in the U.S.</li> </ul>		

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) ★							
Summarize, represent, and interpret data on two categorical and quantitative variables							
Standards	Label	Mathematical Practices	Explana	ations and Exam	oles		
Students are expected to:							
HS.S-ID.5. Summarize	*	HS.MP.1. Make sense of	Students	may use spreadshe	ets, graphing cal	culators, a	and statistical software to
categorical data for two	+	problems and persevere in	create fro	equency tables and $\epsilon$	determine associ	ations or tr	ends in the data.
categories in two-way	*	solving them.					
frequency tables. Interpret			Example	:S:			
relative frequencies in the		HS.MP.2. Reason abstractly					
context of the data (including		and quantitatively.	Two-way	y Frequency Table			
joint, marginal, and			A two-wa	ay frequency table is	shown below dis	playing the	e relationship between age
conditional relative		HS.MP.3. Construct viable	and baid	ness. We took a san	nple of 100 male	subjects, a	and determined who is or is
frequencies). Recognize		arguments and critique the	not daid.	We also recorded tr	he age of the mai	e subjects	by categories.
possible associations and		reasoning of others.				1	1
trends in the data.		HS MR 4 Model with	Dala	I wo-way Fred	uency lable	Tatal	
Connections:		mathematics	Baid	Age	) 45 or older	Iotai	
ETHS-S1C2-01		mainemailes.	Na	Younger than 45	45 or older	4.0	
ETHS-S6C2-03		HS MP 5. Use appropriate	INO Vec	30	11	40	
11-12 BST 9		tools strategically	Tetel	24	30	54	
11-12.WHST.1a-1b:		toole strategioally.	Total	59	41	100	1
11-12.WHST.1e		HS.MP.8. Look for and	The tota	I wave and tatal calve	na antriaa in tha t		we want the area waised
Introduction to		express regularity in repeated	frequence	row and <i>total</i> colum	the body of the te	able above	report the marginal
Statistics 11 2		reasoning.	requenc	les, while entries in i	the body of the ta	ible are the	a joint frequencies.
				v Dolotivo Eroquon			
			The role	y neialive Frequein	body of the teh		ad conditional relative
			frequenc		le bouy of the tab	le ale calle	
			nequenc	163.			
				Two-way Belative	Frequency Tabl	e	
			Bald	Aqe	<u></u>	Total	
				Younger than 45	45 or older		
			No	0.35	0.11	0.46	
			Yes	0.24	0.30	0.54	
			Total	0.59	0.41	1.00	

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) ★						
Summarize, represent, and i	Summarize, represent, and interpret data on two categorical and quantitative variables					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.S-ID.6. Represent data	*	HS.MP.2. Reason	The residual in a regression model is the difference between the observed and			
on two quantitative variables	+	abstractly and	the predicted. Students may use spreadsheets, graphing calculators, and			
on a scatter plot, and	(	quantitatively.	statistical software to represent data, describe how the variables are related, fit			
describe how the variables		-1	functions to data, perform regressions, and calculate residuals			
are related.		HS MP 3 Construct viable	renerious to data, perform regressions, and careatate residuals.			
Connections:		arguments and critique the	Example:			
SCHS-S1C2-05:		reasoning of others	Measure the wrist and neck size of each person in your class and make a			
SCHS-S1C3-01;		reasoning of others.	scatterplot. Find the least squares regression line. Calculate and interpret the			
ETHS-S1C2-01;			correlation coefficient for this linear regression model. Graph the residuals			
ETHS-S1C3-01;		HS.MP.4. Model with	and evaluate the fit of the linear equations.			
ETHS-S6C2-03		mathematics.				
Introduction to						
Statistics 10.1, 10.2		HS.MP.5. Use appropriate				
a. Fit a function to the data;	*	tools strategically.				
use functions fitted to	+					
the context of the data	*	HS.MP.7. Look for and				
Use given functions or		make use of structure.				
choose a function						
suggested by the context.		HS.MP.8. Look for and				
Emphasize linear,		express regularity in				
quadratic, and		repeated reasoning.				
exponential models.						
Connection: 11-12 BST 7						
Introduction to						
Statistics 10.1 10.2						
Introduction to Statistics 10.1, 10.2						

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) +			
Summarize, represent, and	interpret	data on two categorical and o	juantitative variables
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
b. Informally assess the fit	*		
of a function by plotting	+		
and analyzing residuals.	*		
Connections: 11-12.RST.7; 11-12.WHST.1b-1c Introduction to Statistics 10.2 c. Fit a linear function for a	*		
a linear association	+		
Connection: 11-12.RST.7 Introduction to Statistics 10.1, 10.2	*		

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) ★					
Interpret linear models					
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:					

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) ★					
Interpret linear models					
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Connections: SCHS-S5C2-01; ETHS-S1C2-01; ETHS-S6C2-03; 9-10.RST.4; 9-10.RST.7; 9-10.WHST.2f Intermediate Algebra 8.2 Introduction to Statistics 10.1, 10.2	9-10 ★	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> </ul>	Students may use spreadsheets or graphing calculators to create representations of data sets and create linear models.Example: <ul><li>Lisa lights a candle and records its height in inches every hour. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.6), (3, 14.9), (4, 13.2), (5, 11.5), (7, 8.1), (9, 4.7), and (10, 3). Express the candle's height (<i>h</i>) as a function of time (<i>t</i>) and state the meaning of the slope and the intercept in terms of the burning candle.Solution:$h = -1.7t + 20$ Slope: The candle's height decreases by 1.7 inches for each hour it is burning. Intercept: Before the candle begins to burn, its height is 20 inches.</li></ul>		
HS.S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit. Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.5; 11-12.WHST.2e Introduction to Statistics 10.5	*	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.8. Look for and express regularity in repeated reasoning.	<ul> <li>Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients.</li> <li>Example: <ul> <li>Collect height, shoe-size, and wrist circumference data for each student. Determine the best way to display the data. Answer the following questions: Is there a correlation between any two of the three indicators? Is there a correlation between all three indicators? What patterns and trends are apparent in the data? What inferences can be made from the data?</li> </ul> </li> </ul>		

Statistics and Probability: Interpreting Categorical and Quantitative Data (S-ID) *				
Interpret linear models				
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.S-ID.9. Distinguish between correlation and causation.	9-10 ★	HS.MP.3. Construct viable arguments and critique the reasoning of others.	Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment.	
Connection: 9-10.RST.9		HS.MP.4. Model with mathematics.	Example:	
Introduction to Statistics 10.1, 10.2		HS.MP.6. Attend to precision.	• Diane did a study for a health class about the effects of a student's end-of- year math test scores on height. Based on a graph of her data, she found that there was a direct relationship between students' math scores and height. She concluded that "doing well on your end-of-course math tests makes you tall." Is this conclusion justified? Explain any flaws in Diane's reasoning.	

Statistics and Probability: Making Inferences and Justifying Conclusions (S-IC) *				
Understand and evaluate ra	ndom prc	cesses underlying statistical e	experiments	
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples	
Students are expected to:				
HS.S-IC.1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that	*	HS.MP.4. Model with mathematics. HS.MP.6. Attend to precision.		
population. Basic Math/Pre Algebra 8.3 Introduction to Statistics 1.1, 1.3, 1.4, 7.1, 7.2, 8.1, 8.2, 8.3, 8.4, 9.1 - 9.7				

Statistics and Probability: Making Inferences and Justifying Conclusions (S-IC) *					
Understand and evaluate random processes underlying statistical experiments					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	9-10 ★	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> </ul>	Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials. The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process. Example:		
Connections: ETHS-S6C2-03; 9-10.WHST.2d; 9-10.WHST.2f College Algebra 9.3 Introduction to Statistics 4.1		<ul> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and</li> </ul>	<ul> <li>Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability?</li> </ul>		
		express regularity in repeated reasoning.			

Statistics and Probability: Making Inferences and Justifying Conclusions (S-IC) *					
Make inferences and justify conclusions from sample surveys, experiments, and observational studies					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Connections: 11-12.RST.9; 11-12.WHST.2b Introduction to Statistics 1.3, 1.4, 1.5, 10.4	+ *	HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.6. Attend to precision.	Students should be able to explain techniques/applications for randomly selecting study subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment. In statistics, an observational study draws inferences about the possible effect of a treatment on subjects, where the assignment of subjects into a treated group versus a control group is outside the control of the investigator (for example, observing data on academic achievement and socio-economic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment.		
HS.S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	+ (	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.4. Model with mathematics.	Students may use computer generated simulation models based upon sample surveys results to estimate population statistics and margins of error.		
ETHS-S6C2-03; 11-12.RST.9; 11-12.WHST.1e Introduction to Statistics 8.1 - 8.3		HS.MP.5. Use appropriate tools strategically.			

Statistics and Probability:	Making	Inferences and Justifying Co	onclusions (S-IC) *			
Make inferences and justify of	Make inferences and justify conclusions from sample surveys, experiments, and observational studies					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.S-IC.5. Use data from a	+	HS.MP.1. Make sense of	Students may use computer generated simulation models to decide how likely it is			
randomized experiment to	*	problems and persevere in	that observed differences in a randomized experiment are due to chance.			
compare two treatments;		solving them.	Treatment is a term used in the context of an experimental design to refer to any			
use simulations to decide if		HS.MP.4. Model with	prescribed combination of values of explanatory variables. For example, one wants			
differences between		mathematics.	to determine the effectiveness of weed killer. Two equal parcels of land in a			
parameters are significant.			neighborhood are treated; one with a placebo and one with weed killer to determine			
		HS.MP.5. Use appropriate	whether there is a significant difference in effectiveness in eliminating weeds.			
Connections:		tools strategically.				
ETHS-S6C2-03;		HS.MP.8. Look for and				
11-12.RST.4; 11-12.RST.5;		express regularity in repeated				
11-12.WHST.1e		reasoning.				
Introduction to						
Statistics 9.5, 9.6, 10.4						

<b>Statistics and Probability:</b>	Making	nferences and Justifying Co	onclusions (S-IC) *
Make inferences and justify	conclusic	ns from sample surveys, expe	eriments, and observational studies
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.S-IC.6. Evaluate reports based on data.	9-10 ★	HS.MP.1. Make sense of problems and persevere in solving them.	Explanations can include but are not limited to sample size, biased survey sample, interval scale, unlabeled scale, uneven scale, and outliers that distort the line-of-best-fit. In a pictogram the symbol scale used can also be a source of distortion.
Connections: 11-12.RST.4; 11-12.RST.5; 11-12.WHST.1b; 11-12.WHST.1e Basic Math/Pre Algebra 8.1 Introduction to Statistics 2.1 - 2.4, 8.5, 9.7, 10.5		<ul> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	<ul> <li>As a strategy, collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed.</li> <li>Example: <ul> <li>A reporter used the two data sets below to calculate the mean housing price in Arizona as \$629,000. Why is this calculation not representative of the typical housing price in Arizona?</li> <li>King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}</li> <li>Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000}</li> </ul> </li> </ul>

Statistics and Probability: Conditional Probability and the Rules of Probability (S-CP) *					
Understand independence and conditional probability and use them to interpret data					
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples		
Students are expected to:					

Statistics and Probability:	Conditio	onal Probability and the Rule	es of Probability (S-CP) *
Understand independence a	nd condi	tional probability and use ther	n to interpret data
Standards	<u>Labei</u>	<u>Mathematical Practices</u>	Explanations and Examples
Students are expected to.		HS MD 2 Bassan abstractly	Interpretien: The interpretion of two acts A and P is the act of elements that are
AS.S-CF.T. Describe events	**	and quantitatively	<u>Intersection</u> . The <b>intersection</b> of two sets A and B is the set of elements that are common to both set A and set B. It is denoted by $A \cap B$ and is read 'A intersection
(the set of outcomes) using	×	and quantitativery.	R'
characteristics (or categories)		HS.MP.4. Model with	
of the outcomes, or as		mathematics.	• $A \cap B$ in the diagram is $\{1, 5\}$
unions, intersections, or			this means: BOTH/AND
complements of other events		HS.MP.6. Attend to precision.	
("or," "and," "not").			
Connection: 11 10 WHET On		HS.MP.7. Look for and make	
Connection. 11-12.WHS1.2e		use of structure.	
College Algebra 9.4			
Introduction to			
Statistics $1 - 12$			
Statistics 4.1 - 4.5			
			0
			Union: The <b>union</b> of two sets <i>A</i> and <i>B</i> is the set of elements, which are in <i>A</i> or in <i>B</i>
			or in both. It is
			denoted by $A \cup B$ and is read 'A union B'.
			• $A \cup B$ in the diagram is {1, 2, 3, 4, 5, 7}
			• this means: FITHER/OR/ANY
			<ul> <li>could be both</li> </ul>
Evaluations and Examples			High School (0.19)
Omega Math: Standards and Asse	ssment Div	rision	124 Approved date 1.2.13
9-10 = Standards for Grades 9 and	10; 🛠 = A	gebra 2 standards; + = 4 th credit sta	n Geoge plementing the complement at the area And in the area in the set of elements that are

Statistics and Probability: Conditional Probability and the Rules of Probability (S-CP) ★				
Chaerstand Independence a		Methometical Procession	Finite interpret data	
Students are expected to:	Laber	Mathematical Practices	Explanations and Examples	
HS.S-CP.2. Understand that	*	HS.MP.2. Reason abstractly		
two events A and B are	*	and quantitatively.		
independent if the probability				
of A and B occurring together		HS.MP.4. Model with		
is the product of their probabilities, and use this		mathematics.		
characterization to determine		HS.MP.6. Attend to precision.		
if they are independent.				
Connection: 11 12 WHET 10		HS.MP.7. Look for and make		
College Algebra 9 6		use of structure.		
Introduction to				
Statistics 4.3				
HS.S-CP.3. Understand the	*	HS.MP.2. Reason abstractly		
conditional probability of A	*	and quantitatively.		
given b as P(A and b)/P(b),		HS MP 4 Model with		
of A and B as saving that the		mathematics.		
conditional probability of A				
given <i>B</i> is the same as the		HS.MP.6. Attend to precision.		
probability of A, and the		HS MD 7 Look for and make		
given A is the same as the		HS.MF./. LOOK IOF and make		
probability of <i>B</i> .				
Connectioner				
11-12 BST 5				
11-12.WHST.1e				
College Algebra 9.5				
Introduction to				
Statistics 4.3				
_				

Statistics and Probability: Conditional Probability and the Rules of Probability (S-CP) *						
Understand independence a	Understand independence and conditional probability and use them to interpret data					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.S-CP.4. Construct and	*	HS.MP.1. Make sense of	Students may use spreadsheets, graphing calculators, and simulations to create			
interpret two-way frequency	*	problems and persevere in	frequency tables and conduct analyses to determine if events are independent or			
tables of data when two		solving them.	determine approximate conditional probabilities.			
categories are associated		UC MD 0. Dessen shetrestly				
with each object being		HS.MP.2. Reason abstractly				
table as a sample space to		and quantitativery.				
decide if events are		HS MP 3 Construct viable				
independent and to		arguments and critique the				
approximate conditional		reasoning of others.				
probabilities. For example,		3				
collect data from a random		HS.MP.4. Model with				
sample of students in your		mathematics.				
school on their favorite						
subject among math, science,		HS.MP.5. Use appropriate				
and English. Estimate the		tools strategically.				
probability that a randomly						
selected student from your		HS.MP.6. Attend to precision.				
given that the student is in		HS MP 7 Look for and make				
tenth grade Do the same for		use of structure				
other subjects and compare						
the results.		HS.MP.8. Look for and				
		express regularity in repeated				
Connections:		reasoning.				
ETHS-S6C2-03;						
11-12.RST.4; 11-12.RST.9;						
11-12.WHST.1e						
Introduction to						
Statistics 11.2						

Statistics and Probability:	Statistics and Probability: Conditional Probability and the Rules of Probability (S-CP) *					
Understand independence a	Understand independence and conditional probability and use them to interpret data					
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples			
Students are expected to:						
HS.S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. Connections: 11-12.RST.4; 11-12.RST.5; 11-12.WHST.1e College Algebra 9.6 Introduction to	*	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	<ul> <li>Examples:</li> <li>What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced? Are these events independent or dependent?</li> <li>At Johnson Middle School, the probability that a student takes computer science and French is 0.062. The probability that a student takes computer science is 0.43. What is the probability that a student takes French given that the student is taking computer science?</li> </ul>			
Statistics 4.3						

Statistics and Probability: Conditional Probability and the Rules of Probability (S-CP) *					
Use the rules of probability to	Use the rules of probability to compute probabilities of compound events in a uniform probability model				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					

<b>Statistics and Probability:</b>	Conditio	onal Probability and the Rule	es of Probability (S-CP) ★
Use the rules of probability to	o comput	e probabilities of compound e	vents in a uniform probability model
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.S-CP.6. Find the conditional probability of <i>A</i> given <i>B</i> as the fraction of <i>B</i> 's outcomes that also belong to <i>A</i> , and interpret the answer in terms of the model.	*	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.4. Model with mathematics.	Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.
Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.9; 11-12.WHST.1b; 11-12.WHST.1e; College Algebra 9.5 Introduction to Statistics 4.3		HS.MP.5. Use appropriate tools strategically. HS.MP.7. Look for and make use of structure.	
HS.S-CP.7. Apply the Addition Rule, $P(A \text{ or } B) =$ P(A) + P(B) - P(A  and  B), and interpret the answer in terms of the model. Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.9 <b>College Algebra 9.4</b> Introduction to Statistics 4.2	*	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	<ul> <li>Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.</li> <li>Example: <ul> <li>In a math class of 32 students, 18 are boys and 14 are girls. On a unit test, 5 boys and 7 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?</li> </ul> </li> </ul>

<b>Statistics and Probability:</b>	Statistics and Probability: Conditional Probability and the Rules of Probability (S-CP) *				
Use the rules of probability to	o comput	e probabilities of compound e	vents in a uniform probability model		
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.S-CP.8. Apply the general Multiplication Rule in a uniform probability model, P(A  and  B) = P(A)P(B A) = P(B)P(A B), and interpret the answer in terms of the model. Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.9 <b>College Algebra 9.5</b> Introduction to Statistics 4.3	*	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically. HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.		
HS.S-CP.9. Use permutations and combinations to compute probabilities of compound events and solve problems. Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.9 <b>College Algebra 9.4</b> Introduction to Statistics 4.4	*	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.7. Look for and make use of structure.</li> </ul>	<ul> <li>Students may use calculators or computers to determine sample spaces and probabilities.</li> <li>Example: <ul> <li>You and two friends go to the grocery store and each buys a soda. If there are five different kinds of soda, and each friend is equally likely to buy each variety, what is the probability that no one buys the same kind?</li> </ul> </li> </ul>		

Statistics and Probability:	Using P	robability to Make Decisions	s (S-MD) ★	
Standards	l abel	Mathematical Practices	Explanations and Examples	
Students are expected to:	20001			
HS.S-MD.1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. Connections: ETHS-S6C2-03; 11-12.RST.5; 11-12.RST.9; 11-12.WHST.1b; 11-12.WHST.1e Introduction to Statistics 5.1, 5.2, 5.3	★ ★	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	Students may use spreadsheets, graphin represent data in multiple forms. Example: • Suppose you are working for a co- wants to ensure that the home m She asks you to research the size better inform the floor plans of the Solution: • A possible solution could be the m forms. In this case, the results of graph. The student has defined th per household. People per Household 1 2 3 4 5 6 7 9 0.6 0.5 0.4 0.3 0.2 0.1 0.2 0.1 0.2 0.1 0.2 0.2 0.1 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2	g calculators and statistical software to ontractor who is designing new homes. She odels match the demographics for the area. e of households in the region in order to e home. result of research organized in a variety of the research are shown in a table and heir variable as x as the number of people Proportion of Households 0.026 0.031 0.132 0.567 0.181 0.048 0.015

Statistics and Probability: Using Probability to Make Decisions (S-MD) ★								
Standarde	na use tr	Mathematical Practices	Evolanativ	Evaluations and Examples				
Students are expected to:	Lavei	Mathematical Fractices			ampies			
HS.S-MD.2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	∻ + ★	HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Students may use spreadsheets or graphing calculators to complete calculations or create probability models. The expected value of an uncertain event is the sum of the possible points earned multiplied by each points' chance of occurring.					
Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.3; 11-12.RST.4; 11-12.RST.9		HS.MP.6. Attend to precision. HS.MP.7. Look for and make use of structure.	<ul> <li>Example:</li> <li>In a game, you roll a six sided number cube numbered with 1, 2, 3, 4, 5 and 6. You earn 3 points if a 6 comes up, 6 points if a 2, 4 or 5 comes up, and nothing otherwise. Since there is a 1/6 chance of each number coming up, the outcomes, probabilities and payoffs look like this:</li> </ul>					
College Algebra 9.4				Outcome	Probability	Points		
Statistics 5.1, 5.2, 5.3				1	1/6	0 points	_	
				2	1/6	6 points	_	
				3	1/6	0 points	_	
				4	1/6	6 points	-	
				5	1/6	6 points	=	
				6	1/6	3 points	-	
			The expected value is sum of the products of the probability and points earned for each outcome (the entries in the last two columns multiplied together): $\left(\frac{1}{6}\right) \bullet 0 + \left(\frac{1}{6}\right) \bullet 6 + \left(\frac{1}{6}\right) \bullet 0 + \left(\frac{1}{6}\right) \bullet 6 + \left(\frac{1}{6}\right) \bullet 6 + \left(\frac{1}{6}\right) \bullet 3 = 3.50 \text{ points}$					

Statistics and Probability: Using Probability to Make Decisions (S-MD) ★					
Calculate expected values a	Calculate expected values and use them to solve problems				
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples		
Students are expected to:					
HS.S-MD.3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.RST.3; 11-12.RST.9; 11-12.WHST.1b; 11-12.WHST.1e Introduction to Statistics 5.1, 5.2, 5.3		<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.7. Look for and make use of structure.</li> </ul>	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.		
11-12.WHST.1e Introduction to Statistics 5.1, 5.2, 5.3					

Statistics and Probability: Using Probability to Make Decisions (S-MD) ★			
Calculate expected values and use them to solve problems			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.S-MD.4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? Connections: ETHS-S1C2-01; ETHS-S6C2-03; 11-12.WHST.1b; 11-12.WHST.1e Introduction to	* + (	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.7. Look for and make use of structure.</li> </ul>	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.
Statistics 5.1, 5.2, 5.3			

Statistics and Probability: Using Probability to Make Decisions (S-MD) (			
Use probability to evaluate outcomes of decisions			
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.S-MD.5. Weigh the possible outcomes of a	- *	HS.MP.1. Make sense of problems and persevere in	Different types of insurance to be discussed include but are not limited to: health, automobile, property, rental, and life insurance.
decision by assigning	+ •	solving them.	
probabilities to payoff values and finding expected values.	~	HS.MP.2. Reason abstractly	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, guadratic or exponential
		and quantitatively.	functions.
Connections: SSHS-S5C2-			
03, SSHS-S5C5-03, SSHS-		HS.MP.3. Construct viable	
S5C5-05, ETHS-S1C2-01 FTHS-S6C2-03		reasoning of others	
a. Find the expected payoff	*	HS.MP.4. Model with	
for a game of chance.	+	mathematics.	
expected winnings from a	*	HS.MP.5. Use appropriate	
state lottery ticket or a		tools strategically.	
game at a fast-food			
restaurant.		HS.MP.6. Attend to precision.	
Connections: 11-12.BST 3		HS.MP.7. Look for and make	
11-12.RST.9;		use of structure.	
11-12.WHST.1b;			
11-12.WHST.1e		HS.MP.8. Look for and	
Introduction to		express regularity in repeated	
Statistics 5.1		reasoning.	

Statistics and Probability: Using Probability to Make Decisions (S-MD) (				
Use probability to evaluate o	Use probability to evaluate outcomes of decisions			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
<ul> <li>Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.</li> </ul>				
Connections: 11-12.RST.3; 11-12.RST.9; 11-12.WHST.1b; 11-12.WHST.1e Introduction to Statistics 5.1				

Statistics and Probability: Using Probability to Make Decisions (S-MD) (			
Use probability to evaluate outcomes of decisions			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.S-MD.6. Use probabilities	+	HS.MP.1. Make sense of	Students may use graphing calculators or programs, spreadsheets, or computer
to make fair decisions (e.g.,	*	problems and persevere in	algebra systems to model and interpret parameters in linear, quadratic or exponential
drawing by lots, using a		solving them.	functions.
random number generator).		HS MP 2 Passon obstractly	
Connections:		and quantitatively	
ETHS-S1C2-01:		and quantitativoly.	
ETHS-S6C2-03;		HS.MP.3. Construct viable	
11-12.RST.3; 11-12.RST.9;		arguments and critique the	
11-12.WHST.1b;		reasoning of others.	
11-12.WHST.1e			
Introduction to		HS.MP.4. Model with	
Statistics 4.1 - 4.4		mainematics.	
		HS.MP.5. Use appropriate	
		tools strategically.	
		Č, j	
		HS.MP.7. Look for and make	
		use of structure.	

Statistics and Probability: Using Probability to Make Decisions (S-MD) (			
Use probability to evaluate outcomes of decisions			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
HS.S-MD.7. Analyze	*	HS.MP.1. Make sense of	Students may use graphing calculators or programs, spreadsheets, or computer
decisions and strategies	+	problems and persevere in	algebra systems to model and interpret parameters in linear, quadratic or exponential
using probability concepts	*	solving them.	functions.
(e.g., product testing, medical		HS MP 2 Bosson abstractly	
doalie at the end of a dame)		and quantitatively	
gouilo at the one of a game).			
Connections:		HS.MP.3. Construct viable	
ETHS-S1C2-01;		arguments and critique the	
ETHS-S6C2-03		reasoning of others.	
Introduction to		LIC MD 4 Madal with	
Statistics 8.1 - 8.4, 9.1 -		HS.MP.4. Model with	
9.6			
		HS.MP.5. Use appropriate	
		tools strategically.	
		HS.MP.7. Look for and make	
		use of structure.	

### High School – Contemporary Mathematics Overview

### **Discrete Mathematics (CM-DM)**

• Understand and apply vertex-edge graph topics

### **Mathematical Practices (MP)**

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.

#### High School – Contemporary Mathematics *

Discrete mathematics is contemporary mathematics. This area of mathematics is very relevant in today's technologically advanced society. Discrete mathematics provides the underpinnings for many features of the Internet, from encryption of credit card numbers to decompression and compression of photographs, music, and video. It also informs the efficiency of our communication and transportation systems, such as determining the shortest path through a network or identifying the most cost effective design of airline or bus routes. The power of discrete mathematics is exemplified through the motivational impact on students. They are not only immersed in interesting mathematics but are actively engaged in the "doing" of mathematics. Mathematics is not a bystander sport.

Discrete mathematics topics, particularly vertex-edge graphs, afford students the opportunity to access problem solving in a meaningful context. Students strengthen their skills in problem solving, reasoning, conjecturing, communication, analysis, and proof. They apply the Standards for Mathematical Practice as they solve discrete mathematics problems. Discrete mathematics courses play an increasingly important role in the high school curriculum as possible pathways for those students who seek meaningful 4th credit courses that connect to technology and the needs of the 21st century learner.

Graph theory is the formal study of vertex-edge graphs. Unlike graphs used in data analysis, vertex-edge graphs are used to visually represent problem situations. Vertex-edge graphs are used to model and solve problems related to paths, circuits, or the relationship among a set of objects.

**Connections to Modeling.** Mathematical modeling occurs when students follow a multistep process of solving problems and represent the key ideas through a visual representation. These visual representations allow students multiple entry points for solving a problem, ensuring material that is both engaging and accessible. Examples of real word situations that could be modeled using a vertex-edge graph are 1) planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player or 2) engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.

Contemporary Mathematics: Discrete Mathematics (CM-DM) *				
Understand and apply vertex	Understand and apply vertex-edge graph topics			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples	
Students are expected to:				
AZ.HS.CM-DM.1. Study the	+	HS.MP.1. Make sense of	Students may use graphing calculators or computer algebra systems to assist with	
following topics related to	*	problems and persevere in	computations.	
vertex-edge graphs: Euler		solving them.		
circuits, Hamilton circuits, the			Examples:	
Travelling Salesperson		HS.MP.2. Reason abstractly	<ul> <li>A businesswoman in Phoenix is planning a trip to visit clients in Seattle, Los</li> </ul>	
Problem (TSP), minimum		and quantitatively.	Angeles and New York City before returning to Phoenix. The figure below	
weight spanning trees,			gives the cost in dollars of traveling from one city to another. Find the order	
shortest paths, vertex		HS.MP.3. Construct viable	in which these cities should be visited so the total travel cost is at a	
matricos		reasoning of others	minimum.	
mances.		reasoning of others.	Seattle	
Connections:		HS MP 4 Model with	\$1500 NYC	
ETHS-S6C2-03:		mathematics.		
11-12.RST.4; 11-12.RST.5;			0 st 00	
11-12.RST.9;		HS.MP.5. Use appropriate	17 12 4 <u>1</u>	
11-12.WHST.1b;		tools strategically.	44 O 10	
11-12.WHST.1e			\$2.	
		HS.MP.6. Attend to precision.	LA \$50	
Discrete Structures				
chapter 5		HS.MP.7. Look for and make	Prioenix	
•		use of structure.	Note that the businesswoman's trip is the same as a circuit that starts at	
		LIC MD 9. Look for and	vertex 1 (Phoenix), visits each other vertex exactly once, and returns to	
		HS.MP.8. LOOK IOF and	vertex 1. In other words, the circuit is a Hamiltonian circuit, and the	
		reasoning	businesswoman's task is to find the Hamiltonian circuit of least total weight	
		reasoning.	(given the weighted graph).	
			Continued on next page	

Contemporary Mathematics: Discrete Mathematics (CM-DM) *			
Understand and apply verte	x-edge gi	raph topics	
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
Discrete Structures 5.2			Which directed graph below represents a tournament on four vertices, where all players but one are champions?
			Graph 1 Graph 2 Graph 3 Graph 4
			<ul> <li>Build a tournament on 5 vertices where all players but one are champions.</li> <li>Juanita claims that the graph below has an Euler path but not an Euler circuit. Justify her claim.</li> </ul>

Contemporary Mathematics: Discrete Mathematics (CM-DM) *			
Understand and apply vertex-edge graph topics			
<u>Standards</u>	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
AZ.HS.CM-DM.2. Understand, analyze, and apply vertex-edge graphs to model and solve problems	+ ★	HS.MP.1. Make sense of problems and persevere in solving them.	Students may use graphing calculators or computer algebra systems to assist with computations.
related to paths, circuits, networks, and relationships among a finite number of		HS.MP.2. Reason abstractly and quantitatively.	<ul> <li>Find a minimal route that includes every street (e.g., for trash pick up).</li> <li>Find the shortest network connecting specified sites.</li> </ul>
elements, in real-world and abstract settings.		HS.MP.3. Construct viable arguments and critique the reasoning of others.	
Connections: ETHS-S6C2-03; 11-12.RST.9; 11-12.WHST.1b;		HS.MP.4. Model with mathematics.	
11-12.WHST.1e;		HS.MP.5. Use appropriate tools strategically.	
Discrete Structures chapter 5 (5.5)		HS.MP.6. Attend to precision.	
		HS.MP.7. Look for and make use of structure.	
		HS.MP.8. Look for and express regularity in repeated reasoning.	

Contemporary Mathematics: Discrete Mathematics (CM-DM) ★			
Standards	Label	Mathematical Practices	Explanations and Examples
Students are expected to:			
AZ.HS.CM-DM.3. Devise, analyze, and apply algorithms for solving vertex-edge graph	+ ★	HS.MP.1. Make sense of problems and persevere in solving them.	In exploring minimum spanning tree situations students devise, analyze, and apply algorithms as they adopt strategies to confront the problem. Such strategies can lead to Kruskal's algorithm, Prim's algorithm, or the "nearest neighbor" algorithm.
Connections:		HS.MP.2. Reason abstractly and quantitatively.	Students may use graphing calculators or computer algebra systems to assist with computations.
11-12.RST.3; 11-12.RST.4; 11-12.RST.9; 11-12.WHST.1a; 11-12.WHST.1b; 11-12.WHST.1e		HS.MP.3. Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics	<ul> <li>Susan is a city planner in charge of the development of roads for a recreational area. The graph shows locations in the area, the possible roads that could be built between locations, and the cost in thousands of dollars to build each road. Find the smallest possible cost of building enough roads to connect the locations.</li> </ul>
Discrete Structures chapter 5, 6		<ul> <li>mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	<ul> <li>Algorithm to Find a Minimum Spanning Tree in a Connected Graph</li> <li>Given a connected graph with weights on the edges:</li> <li>Step 1. List the edges of the graph by increasing weights.</li> <li>Step 2. Choose the edge with the smallest weight.</li> <li>Step 3. Continue to choose the next edge with the smallest weight as long as choosing that edge does not create a circuit.</li> <li>Step 4. Stop when the result is a spanning tree.</li> </ul> The graph shown is the original graph and also shows the spanning tree (bolded edges) that would be produced by applying the algorithm. The smallest possible cost to build roads connecting all the sites would be to build a road between the theater and restaurant (2), between the restaurant and amusement park (3), between the amusement park and hotel (8), between the hotel and the sports complex (9), and between the sports complex and the museum (10). There is a minimum total cost of \$32,000 to build the roads at the recreational area.
			2 Restaurant 3 Amusement 5 12 Park 5 12 Park 5 9 Hotel 16 Complex 10 Museum

Contemporary Mathematics: Discrete Mathematics (CM-DM) *			
Understand and apply vertex-edge graph topics			
<u>Standards</u>	<u>Label</u>	Mathematical Practices	Explanations and Examples
Students are expected to:			
AZ.HS.CM-DM.4. Extend work with adjacency matrices for graphs, such as interpreting row sums and using the nth power of the adjacency matrix to count paths of length <i>n</i> in a graph. Connections: ETHS-S6C2-03; 11-12.RST.4; 11-12.RST.5; 11-12.RST.9; 11-12.WHST.1a; 11-12.WHST.1b; 11-12.WHST.1b; 11-12.WHST.1e <b>Discrete Structures 5.4</b>	+ *	<ul> <li>HS.MP.1. Make sense of problems and persevere in solving them.</li> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated</li> </ul>	The adjacency matrix of a simple graph is a matrix with rows and columns labeled by graph vertices, with a 1 or a 0 in position ( <b>v</b> _i , <b>v</b> _j ) according to whether <b>v</b> _i and <b>v</b> _j are adjacent or not. A "1" indicates that there is a connection between the two vertices, and a "0" indicates that there is no connection. Students may use graphing calculators or computer algebra systems to assist with computations.
for graphs, such as interpreting row sums and using the nth power of the adjacency matrix to count paths of length <i>n</i> in a graph. Connections: ETHS-S6C2-03; 11-12.RST.4; 11-12.RST.5; 11-12.RST.9; 11-12.WHST.1a; 11-12.WHST.1b; 11-12.WHST.1e <b>Discrete Structures 5.4</b>	*	<ul> <li>HS.MP.2. Reason abstractly and quantitatively.</li> <li>HS.MP.3. Construct viable arguments and critique the reasoning of others.</li> <li>HS.MP.4. Model with mathematics.</li> <li>HS.MP.5. Use appropriate tools strategically.</li> <li>HS.MP.6. Attend to precision.</li> <li>HS.MP.7. Look for and make use of structure.</li> <li>HS.MP.8. Look for and express regularity in repeated reasoning.</li> </ul>	adjacent or not. A "1" indicates that there is a connection between the two vertices, and a "0" indicates that there is no connection. Students may use graphing calculators or computer algebra systems to assist with computations.

Standards for Mathemat	Standards for Mathematical Practice				
<u>Standards</u>		Explanations and Examples			
Students are expected to:	Mathematical Practices are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.				
HS.MP.1. Make sense of problems and persevere in solving them. Met in Geometry, Elementary, Intermediate and College Algebra		High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.			
HS.MP.2. Reason abstractly and quantitatively. Met in Geometry, Elementary, Intermediate and College Algebra		High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.			
## Omega Math[™] Standards Articulated by Grade Level

Standards for Mathematical Practice				
Standards		Explanations and Examples		
Students are expected to:	Mathematical Practices are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.			
HS.MP.3. Construct viable arguments and critique the reasoning of others. Met in Geometry, Elementary, Intermediate and College Algebra		High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.		
HS.MP.4. Model with mathematics. Met in Geometry, Elementary, Intermediate and College Algebra		High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated ituation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose		

## Omega Math[™] Standards Articulated by Grade Level

Standards for Mathematical Practice				
Standards		Explanations and Examples		
Students are expected to:	Mathematical Practices are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.			
HS.MP.5. Use appropriate		High school students consider the available tools when solving a mathematical problem. These		
tools strategically.		tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a		
Met in Geometry, Elementary, Intermediate and College Algebra		High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.		
HS.MP.6. Attend to precision. Met in Geometry, Elementary, Intermediate and College Algebra		High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.		
HS.MP.7. Look for and make use of structure. Met in Geometry, Elementary, Intermediate and College Algebra		By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$ , older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers <i>x</i> and <i>y</i> . High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.		

## Omega Math[™] Standards Articulated by Grade Level

Standards for Mathematical Practice				
<u>Standards</u>		Explanations and Examples		
Students are expected to:	Mathematical Practices are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.			
HS.MP.8. Look for and express regularity in repeated reasoning. Met in Geometry, Elementary, Intermediate and College Algebra		High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$ , $(x - 1)(x2 + x + 1)$ , and $(x - 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.		